Introduction

May 6, 2024 10:48 AM

A market is where financial instruments are traded
- Reduce transaction costs
- Promote efficient allocation of resources
- Determine the prices of financial instruments and assets
- Provide information

Financial instruments
- Monetary contracts between parties that can be traded
- Include equity, debt and derivative instruments
- Financial assets
    - Equity and debt instruments
    - Claims on real assets and/or cash flows generated from real assets
    - Real assets
        - Produce goods or services
        - Physical or non-physical

Resource allocation by financial market (Example 1)
- Corp. A has invented a new product but lack the funds to product
    - Equipment (one time investment, 20 years): $100 million
    - Labor expense: $10 million per year
    - Potential revenue: $20 million per year for 20 years
- Issue new shares worth of $110 million on the financial market
    - Investors purchase the shares @ $110 million
- Corp. A purchases equipment, employ workers, start production, and sells products
    - Pay $100 million for equipment
    - Pays wages of $10 million
    - Collect revenue of $20 million per year, $10 million per year of profit
    - Pay a dividend of $8 million per year back to the investors for 20 years.
- Investors
    - Provide non-productive resources of $110 million for production
    - End up with a profit of $50 million on investment (20 * $8 million-$110 million)
- Workers
    - Receive wage compensation
- Corp. A
    - Earn profits
- Consumers
    - Consume new products, utility increases
- Without a financial market, the above social gains are lost

Types of financial markets (Money versus Capital)
- **Money** markets: for short-term debt instruments
    - Mature within 1 year (very liquid)
    - Treasury bill (T-Bill), commercial paper, etc
        - T-Bill: Short term debt/bond issued by government
        - Pink sheet is stock/bond traded on OTC market
    - Usually organized as **dealer market**
        - Dealer: large financial institution, purchase securities in large quantity and sell to investors.
        - Chartered banks or investment dealers sell securities out of inventory
        - Also referred to as the over-the-counter (OTC) market
    - Liquid: how quickly it can convert to cash

- **Capital** markets: for long-term debt and equity securities
    - Stocks, government/corporate bonds, etc
        - Treasury bonds: long term treasury bond/equity issued by government
    - **Broker/auction market**
        - Investment brokers match buyers and sellers
    - Stock exchange usually organized as an auction market or a hybrid of dealer and auction market

Types of financial markets (Primary versus Secondary)

- **Primary** markets: for newly issued securities
    - Fund flows from investors to the firm (investment banks) to issue new securities
    - Initial Public Offering (IPO): a corporation issues securities to the general public for the first time
    - Usually an **underwriter** is involved
        - A (group of) large investment dealers, called syndicate
        - Advise on issuance structure, assess risk, determine price
        - Purchases the entire issuance at a discounted price, then sell to the general public
        - RBC Dominion, CIBC World Markets, TD Securities Inc.
- **Secondary** markets: for already issued securities
    - Fund flows from one investor to another
    - Improve the liquidity of financial assets
- Comparison:
    - Primary: firm gets money from investors and issue bonds and stocks
    - Secondary: trading among investors

Participants of financial markets

- Households: net suppliers of funds (capital)
- Firms: net demanders of funds (capital)
- Governments: can be either
- Financial intermediaries
    - Commercial banks, investment banks, insurance companies, etc.
    - Match different sides of participants
    - Act as dealers in the financial markets
    - Pool and manage smaller investors' funds
    - Specialization and economies of scale
- Categories of financial intermediaries
    - Borrowing and lending institutions
        - Commercial banks, credit unions
        - Match lenders with borrowers
    - Investment companies
    - Investment banks

Debt instrument (Money market Instruments)

- Treasury bill
    - Issued by the Government of Canada
    - Backed by fiscal power of the government
    - Promise to pay the holder a one-off payment at a given day
    - Short maturities: 30, 60, 90 days, 1/2, 1 year
    - Very liquid
    - Purchased by investment dealers, chartered banks and the Bank of Canada
- Certificate of deposit (CD)
    - Time deposit with a chartered bank cannot be withdrawn on demand
    - A similar kind is GIC (guaranteed investment certificate)
    - Generally non-transferable in Canada
    - Bearer deposit notes (BDNs)
        - CDs in denomination of $100,000+

            - Marketable (allowed to be traded, more liquid)
- CDs and GICs in Canada are insured by the Canada Deposit Insurance Corporation (CDIC)
- Commercial Paper
    - Short term debt, issued by large corporations (matures in 1 to 2 months)
    - Denomination at least $50,000
    - Cheaper than bank credit
    - Money market funds
    - Higher yield than bank deposit (but also higher risk)
    - Rated for credit quality by the major rating agencies
- Repurchases Agreement (Repo)
    - BoC or financial intermediaries sell government securities (debts) and promise to buy them back the next day
    - The difference between sell and repurchase price is the overnight interest rate
    - Backed by government securities
- Reverse Repo
    - Mirror image of Repo
- Eurodollars
    - US dollar-denominated deposits at foreign banks or foreign branches of American banks
    - Time deposit that usually mature in less than six months
    - Not subjected to regulations by the Federal Reserve Board
    - Less liquid, riskier but higher yields than domestic deposit

Long term debt (fixed income) instruments
- Government Bonds
    - Government of Canada bonds
        - Long-term debt securities issued by the Canadian federal government
        - Backed by fiscal power of the Canadian government
        - Varying maturities at issue date, up to 40 years
        - Holders receive a fixed periodical interest payment (monthly, semi-annual, annual), referred to as coupons
    - Provincial and Municipal bonds
        - Typically used for local project financing (bridge, public power station)
        - Backed by tax revenue or project revenue (bridge toll)
- Corporate bonds
    - Issued by corporations (semi-annual coupon)
    - Often bought by insurance companies
    - Actively traded (very liquid)
    - Subject to credit risk
        - Companies go bankrupt
    - Below investment grade entails considerable risk
        - Moody's: BA1 and below
        - Standard and Poor's: BB+ and below
        - Fitch: BB+ and below

Equity
- Ownership contract of the corporation
    - Issued by the corporation
- Common stock
    - Owner can vote on major decisions
- Preferred stock
    - No voting rights
    - Priority in dividend and claims of assets in bankruptcy
- Limited liability
    - Limited to how much I pay. What happened to company won't cost me more than what I paid
- Residual claim

    - Claim assets after other stakeholders: e.g. debt holders
- Separation of ownership and management

Stock market indices

- Track the overall performance of the stock markets
- Two major methods of constructing a stock market index
    - Market-value-weighted index
        - Track investment proportional to market capitalization
        - Market capitalization=share price × shares outstanding
        - $index = \frac{\sum_i (p_i \times \#share_i)}{D}$. (scaled market captialization)
        - p_i= share price of commpany i in the index.
        - Number of shares of company i excludes control block: the block of 20%+of the shares.
        - D= a divisor, some fixed number to set the initial value of the index.
            - This is a random number, independent of the number of companies.
        - e.g.:
            - TSX composite index: 223 largest firms traded on the Toronto Stock Exchange
            - Standard & Poor 500: 500 large firms on the US stock exchange
    - Price-weighted index
        - Track investment in one share of each company included in the index
        - $index = \frac{\sum_i p_i}{D_p}$.
        - D_p= a divisor, some fixed number to set the initial value of the index.
            - It may change over time to adjust for stock splits
            - If stocks in the index incur a stock split (reverse split), a new divisor will be set.
            - The new divisor will be in use until the next time a stock split occurs
            - Stock split: a process of splitting current shares to keep the liquidity of stocks, a single share becomes x shares. The price becomes original price/x.
        - e.g.:
            - Dow Jones Industrial Average (DJIA)
                - 30 blue-chip corporations traded on the New York Stock Exchange (NYSE)
                - Blue-chip: large, stably performing corporations
- Example:

Stock	# shares	Price	Market value (capitalization)
A	200	$80	$16,000=200*$80
B	500	$70	$35,000=500*$70

- Market-value-weighted index.
    - Assume D=100, Index=[$16,000+$35,000]/100=$510
- Price-weighted index
    - Assume D=2, Index=($80+$70)/2=$75.

- Now, if B announces a 2for1 stock split.

Stock	# shares	Price	Market value (capitalization)
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B	1000	$35	$35,000=1000*$35

- Market-value-weighted index.
    - Index=[$16,000+$35,000]/100=$510
    - This is stable and won't change
- Price-weighted index
    - We need to adjust D.
    - Index=($80+$35)/D2=$75 gives D2=115/75.
    - D2 is used for the index until the next stock split

- What matters is not the index at a specific time. We are interested in how the indices change over time.

Derivatives (financial instruments)

- Contractual arrangements between involved parties for the transaction of assets
    - Values are derived from underlying assets (financial assets, commodity, real assets)
    - Derivative itself is also a financial asset
- Futures
    - Obligation to deliver an asset or its cash value on the delivery date at an agreed-upon price
    - Long position: obligated to buy the asset
    - Short position: obligated to deliver the asset
    - e.g.: buy a house that is not built, sign a contract for $1.5 million. After one year, the house is built, and the transaction should complete with $1.5 million. Cannot change the price after the contract is signed and the developer must deliver the house on the specified date. The buyer is in long position. The developer is in short position.
    - Future price may go negative: e.g. Crude oil future in April, 2020
- Options
    - Right to buy or sell an asset on or before expiration date at a specified price
    - Call option: right to buy
    - Put option: right to sell
    - Purchaser has the option not to exercise the right
    - Transaction price (strike price) will not change
    - Price of option can change
- Warrants: similar to call options
    - Stock warrants

Consumption Choice Model

May 8, 2024 11:45 AM

Individuals and institutions have different income patterns and different inter-temporal consumption preferences. Because of this, a market for money arises to coordinate the allocation of wealth (consumption) across time. The price of money (fund) is the interest rate.

Example:

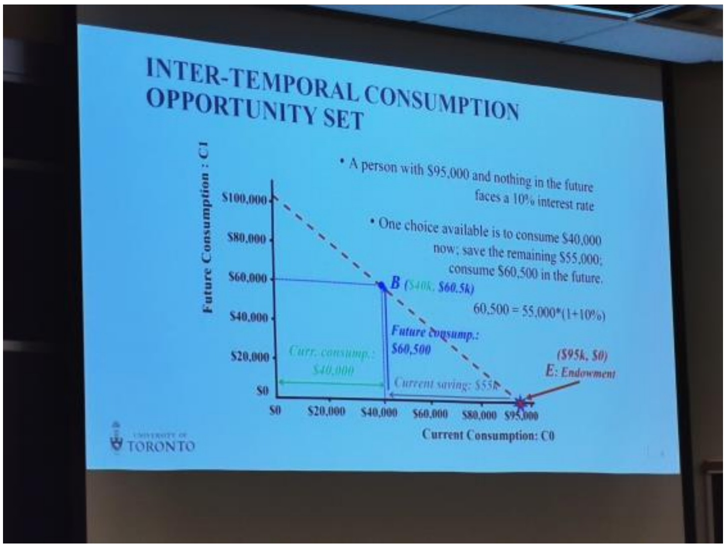
- Consider a dentist who earns $200,000 per year and chooses to consume $80,000 per year. He has $120,000 in surplus money to invest.
- First scenario
  - He could loan $30,000 to each of 4 college seniors. They each promise to pay him back with interest after they graduate in one year.
  - Interest rate: r_0.
- Second scenario
  - Rather than performing the credit analysis 4 times, he could loan the whole $120,000 to a financial intermediary (e.g. a bank) in return for a promise to repay the $120,000 in one year with interest
  - The intermediary in turn loans $30,000 to each of the college
  - Interest rate: r_1.
- $r_0 > r_1$, but in turn the risk is smaller with a bank.
- Bank:
  - Reduce transaction cost
    - Save time and money to find the students and collect payments
  - Shift and control credit risk (risk sharing)
    - Screen for who are likely to pay back the loan
    - Better monitor and control credit risk
    - Guarantee payment to the dentist, shifts risk away from the dentist

Financial intermediaries

- Size intermediation
  - Take a large loan from one party and make small loan to multiple parties
- Term intermediation
  - Commercial banks finance long-term mortgages with short-term deposits
- Risk intermediation
  - Tailor the risk characteristics of securities for borrowers and lenders with different degrees of risk tolerance

Consumption choice over time

- An individual can alter their consumption pattern across time through borrowing and saving (lending)
  - Borrowing in the form of trading of instruments
- Assumption: individuals would like to smooth consumptions across time
  - Modeled by the indifference curves over consumption across time
- We can illustrate this by graphing

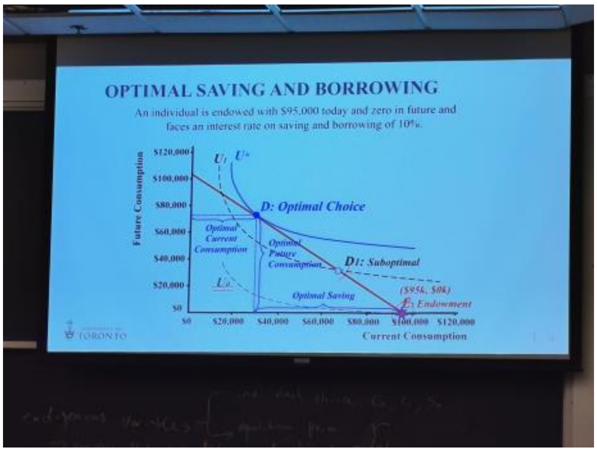
- 

    - The 10% interest rate means the person is able to borrow or lend at 10%
    - In general, C0: present value, C1: future value
    - Endowment ($95k, 0): $95k is current endowment, $0 is future endowment
    - y-intercept: consume nothing today, will get $95k(1+10%) in the future.
- Mathematically,
    - Horizontal axis: consumption in the current period
        - Total amount of money that an individual can spend on current consumption
        - Money does not yield utility, consumption does; model measures consumption using money
    - Vertical axis: consumption in the future period
    - Slope: the (gross) return on savings
        - Save $1 today, the future value will be $(1+r).
    - Current consumption: C0
    - Future consumption: C1
    - Initial endowment: (W0, W1)
        - W0: current endowment
        - W1: future endowment
    - Opportunity set: C1=W1+(1+r)(W0-C0)
        - W0-C0: Current saving S0. If positive, it is saving (landing). If negative, it is borrowing
        - (1+r)(W0-C0): gross return on saving
    - Future value form: C1=-(1+r)C0+[(1+r)W0+W1].
    - Present value form: C0+C1/(1+r)=W0+W1/(1+r).
        - Life time consumption = Life time wealth

Endogenous variables: variables that are determined by a model
- Individual choices: C0, C1, S0
- Equilibrium price

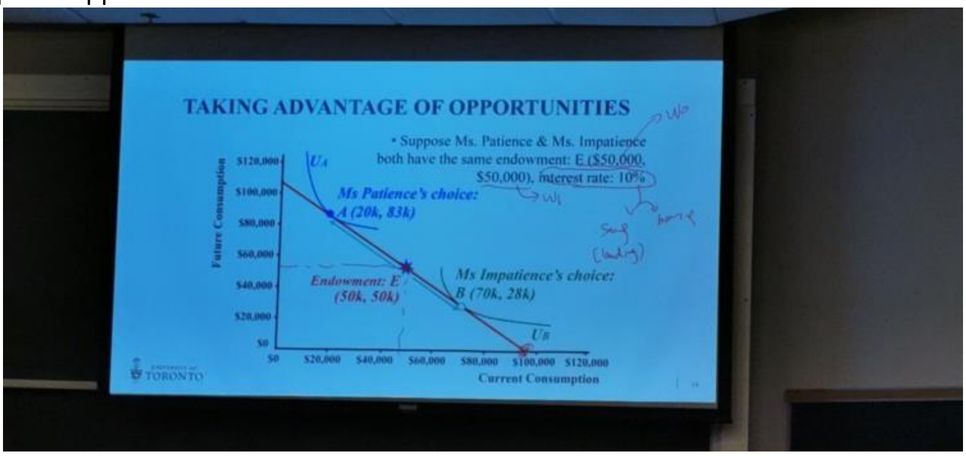
Exogenous variables: variables that are given by the setup and cannot be changed by a model

How much to save or borrow
- Initial endowment
- Interest rates on saving and borrowing

- 

- Financial market and instruments help individuals to smooth consumption over-time.

Example of opportunities:



- Maximum borrow (borrowing limit): W1/(1+r)=50k/1.1=45.5k
  - W1 is the maximum dollar that they can pay tomorrow
  - They cannot borrow so much that they consume 0 for C1
- Ms. Patience: W0=50k, C0=20k (Net saver)
  - S0=50k-20k=30k: lend money, providing/supplying fund
- Ms. Impatience: W0=50k, C0=70k (Net borrower)
  - S0=50k-70k=-20k: borrowing money, demanding funds
- Demand doesn't match supply, more supply than demand (Excess supply).
  - Price (interest rate) is too high
- Drops interest rate to 5% to clear the market, but endowment is exogenous, and won't change
  - Ms. Patience's choice: (25k, 76.25k)
  - Ms. Impatience's choice: (75k, 23.75k)
  - Net saving=net borrowing

Market clearing
- Market is cleared when the supply and demand are balanced as price adjusts
- The equilibrium price is the price that equates supply with demand

Competitive market
- Many traders or investors, no individual can move market prices
- No transaction costs
- Perfect information (information about borrowing and lending is available)
- There can be only one equilibrium interest rate, otherwise arbitrage opportunities would arise

Arbitrage
- The process of making risk-free profit through exploiting price differences
- Suppose there are two equilibrium interest rates in the market: one high rate and one low rate
  - Investors can borrow money from the bank offering low rate and save at the bank offering high rate

        - Investors don't need any endowment to do this
    - Using the higher interest collected from saving to pay back the lower interest from borrowing: risk-free profit
    - Everyone tends to do the same, then
        - Excess demand for funds in the low-rate bank: bid up the low interest rate
            - Higher demand, higher price, increase rate
        - Excess supply in the high-rate bank: push down the high interest rate
            - Higher supply, lower price, decrease rate
        - The two interest rates will converge
- Arbitrage window closes soon

Basic principle of investment

- An investment must be at least as desirable as the opportunities available in the financial markets
- The investment should have as high return as returns in the financial markets

Example:

- Consider an investment opportunity that costs $50k this year and provides a certain cash flow of $54k next year.
    - Certain: means risk-free, there is no risk premium
- Whether or not this is a good deal depends on the interest rate available in the financial markets
- Rate of return on the investment project: $54k=$50k(1+r), r=8%
    - If the financial markets cannot offer a lending rate of 8% or above, the project is worth investing

Example:

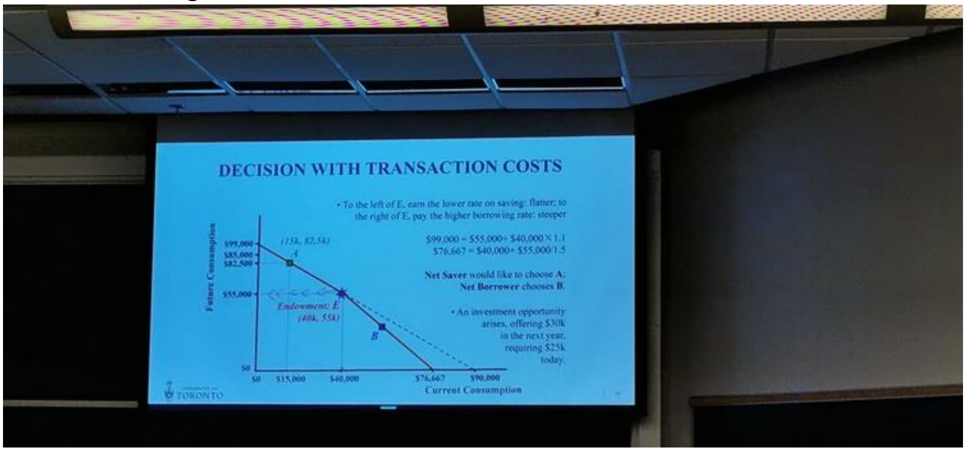
- Consider an investor who has an initial endowment of income of $40k, and $55k next year. Suppose that she faces a 10% risk free interest rate from financial market on saving and borrowing, and is offered the following investment: $25k investment, certain cash flow of $30k next year
- Rate of return on the investment project: $30k=$25k(1+r), r=20%
- Endogenous factors: C1, C2, S0, whether to invest.
- Case 1 (Consume $15k now):
    - Save $25k with 10% interest rate, consume $25k*1.1+$55k=$82.5k next year
    - Invest $25k in a project and earn 20% rate of return, consume $85k next year (better)
- Case 2 (Consume $30k now):
    - Note that the person doesn't have $25k now. Take $10k from current endowment and borrow $15k from the bank for investment in the current year. Then next year, she will receive $30k from the investment and pay 1.1*$15k=$16.5k to the bank. With future endowment of $55k+($30k-$16.5k)=$68.5k
    - If pure saving, consume $10k*1.1+$55k=$66k.
- The outside investment opportunity shifts the consumption opportunity upwards. With proper arrangements, individual can now consume at any point on the shifted line. The amount of shift is the difference between rate of return on outside investment and the rate offered in the financial market
    - (0.2-0.1)*25k=$2.5k

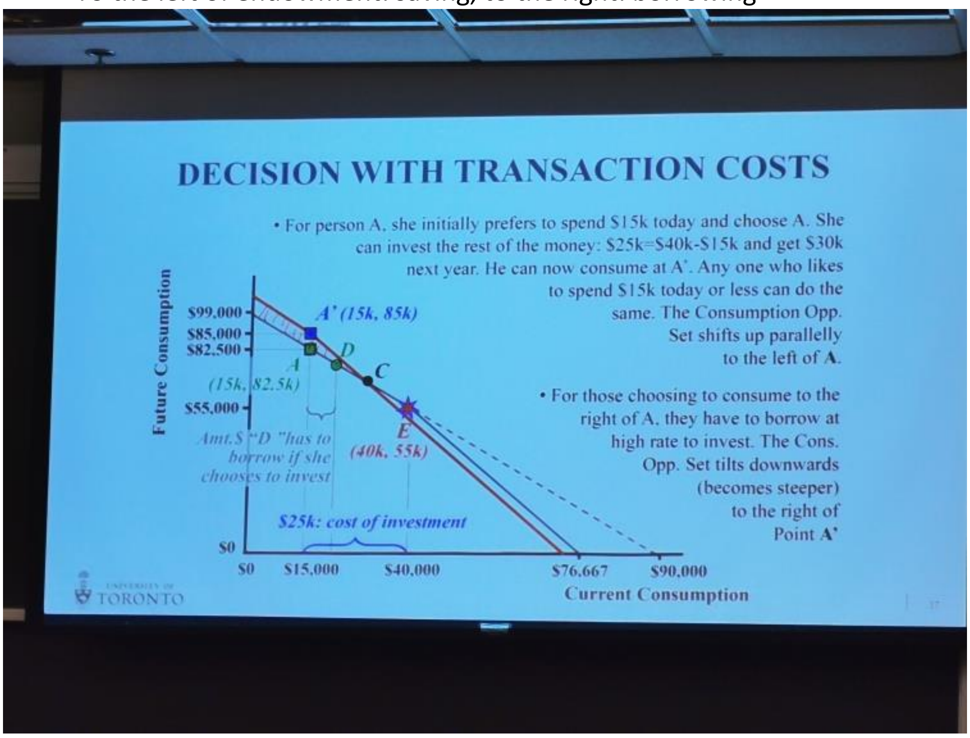
Fisher separation theorem

- The best investment decision is separated from individual's consumption decisions, regardless of their personal preferences
- Separating investment decision-making from the shareholders is a basic requirement of the modern corporation
    - The best investment decision made by the corporation will reach consensus among all the shareholders
    - This decision will benefit all the shareholders as it will expand the consumption opportunity set.

- The validity of the theorem relies on the assumption of a competitive capital market
  - No transaction cost
  - Perfect information
  - All investors are price takers
- If any of the above assumptions are violated, different individuals will arrive at different investment decisions
  - If there exist transaction costs (need to spend resources to match lenders with borrowers), saving (lending) rate < borrowing rate. Banks earn difference to compensate their costs

Decision with transaction costs

- Endowment: $40k today, $55k next year, investment opportunity pays $30k next year and requires $25k today
- Interest rate on saving (lending) is not equal to the rate of borrowing
  - Saving earns 10%
  - Borrowing 50%
- 

  - To the left of endowment: saving, to the right: borrowing
- 

  - To the left of point C, consumption opportunity set expands. They will take the investment
  - To the right of point C, consumption opportunity set is reduced. They will not take the investment
  - Fisher separation theorem breaks down. People don't agree on investment decision

Time Value of Money

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Time value of money

- The value of one dollar differs at different points in time
- Money grows if
  - Interest rate on saving is positive
  - Other investment opportunities yield positive returns
- The gross payoff (principal + interest) of money differs at different points in time
- e.g.
  - A: get \$10k today
  - B: get \$10k 10 years from today
  - A might be better, because of inflation, return from investment, future exchange rate (riskiness), time preference, opportunity cost of cash, etc. Everything can be modelled by rate of return
- We should not evaluate money or any cash flow without referring to time

Future value: one period

- Invest \$10k at 5% interest for one year. Investment grows up to \$10.5k
  - Present value (PV): \$10k, the value we set aside today
- The total amount at the end of the investment is called the **future value (FV)**.

Future value: multi period

- $FV = C_0 \times (1+r)^T$.
  - C_0: cash flow at time 0.
  - r: constant interest rate
  - T: number of periods over which the cash is invested
- Assumptions
  - Investment yields payoff at the end of each period
  - The principal and the payoff is immediately reinvested at the same rate of return

Time of doubling

- Deposit \$5k today in an account paying 10% per year, how long does it take to grow to \$10k?
- $10000 = 5000(1+0.1)^T$, gives $T = \frac{\ln 2}{\ln 1.1} = 7.27$ year.
- Rule of 72: the time (number of periods) to double the initial investment is approximately: $\frac{72}{100r}$.

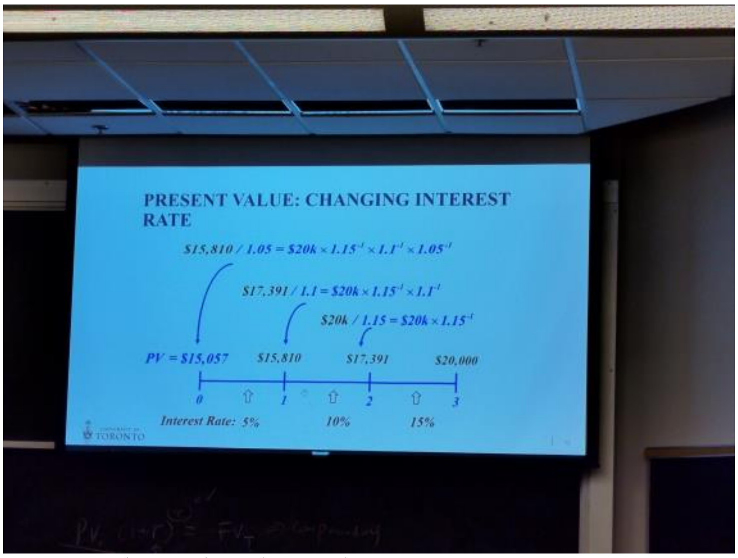
Present value: one period

- If we are promised of \$10k due in one year and the interest rate is 5%, this payment is worth $\frac{\$10k}{1.05} = \$9523.81$ in today's dollars.
- The amount that a borrower would need to set aside today to be able to meet the promised payment of \$10k in one year is called the **present value (PV)** of \$10k.

Present value: multi period

- $PV = \frac{FV}{(1+r)^T}$: discounting future value.

Present/future value with changing interest rate

- 

- $FV = PV(1 + r_1) \cdots (1 + r_T)$.

Compare money at different times

- \$300 now and \$500 in 5 years, which is more valuable?
- It depends on the rate of return on our savings over the 5 years
- Comparison must be made at a certain point of time
- Traditionally, use present time (time zero): compare PV
    - Time zero is usually the time when investment decision has to be made
- Can also use 5 years from now: compare FV
- Theoretically, can use any point of time to compare, but need to correctly convert payments to PV or FV correspondingly
- If the interest rate on saving is 10% per year, using PV
    - \$300 is \$300.
    - PV of \$500 is $\frac{500}{1.1^5} = 310.46$. (better)
- If the interest rate is 20% per year,
    - PV of \$500 is $\frac{500}{1.2^5} = 200.93$. (worse)
    - 20% is the opportunity cost that we would have earned

Simplified cash flows

- Perpetuity:
    - A constant stream of cash flows that lasts forever
    - $PV = \frac{C}{1+r} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \cdots$. (The value at one period before the first payment)
    - The formula for the present value of a perpetuity is $PV = \frac{C}{r}$.
    - E.g. if you want to provide an income stream of \$150 per year, how much do you have to save in the bank today, if the interest rate is 10% per year?
        - $PV = \frac{150}{10\%} = \$1500$.
- Growing perpetuity:
    - A stream of cash flows that grows at a constant rate forever
    - $PV = \frac{C}{1+r} + \frac{C(1+g)}{(1+r)^2} + \frac{C(1+g)^2}{(1+r)^3} + \cdots$. (The value at one period before the first payment)
    - The value is $PV = \frac{C}{r-g}$, infinite if $g \geq r$.
    - E.g. if the expected dividend next year is \$1.3, and grow at 5% per year, discount rate 10%, then
        - $\text{PV} = \frac{\$1.3}{0.1-0.05} = \$26$.
- Annuity:
    - A stream of constant cash flows that lasts for a fixed number of periods.
    - $PV = \frac{C}{(1+r)} + \cdots + \frac{C}{(1+r)^T}$.

- The value is $PV = \frac{C}{r}\left(1 - \frac{1}{(1+r)^T}\right)$.
  - T is the number of payments, r is the discount rate, C is the first payment.
- E.g. if you can afford a $400 monthly payment, how much can you afford if the monthly interest rate is 7/12% over 36-month loan? $PV = \frac{400}{\frac{7}{12}\%}\left(1 - \frac{1}{\left(1+\frac{7}{12}\%\right)^{36}}\right) = \$12{,}954.59$.
- Delayed payment: what is the PV of a four-year annuity of $100 per year that makes its first payment two years from today if the discount rate is 9%?
  - Compute PV at one period before first payment (year 1) $PV_1 = \frac{100}{9\%}\left(1 - \frac{1}{(1+9\%)^4}\right) = \$323.97$.
  - $PV_0 = \frac{323.97}{1.09} = \$297.22$.

- Growing annuity:
  - A stream of cash flows that grows at a constant rate for a fixed number of periods
  - $PV = \frac{C}{(1+r)} + \frac{C(1+g)}{(1+r)^2} + \cdots + \frac{C(1+g)^{T-1}}{(1+r)^T}$.
  - The value is $PV = \frac{C}{r-g}\left(1 - \left(\frac{1+g}{1+r}\right)^T\right)$.
    - g is growth rate of payments, r is the discount rate.
  - e.g. first year rent: $8500, grow at 7% per year, income stream over the first 5 year if the discount rate is 12%:
    - $PV = \frac{8500}{12\%-7\%}\left(1 - \left(\frac{1+7\%}{1+12\%}\right)^5\right) = \$34706.26$.
  - E.g. James plans to purchase a house worth $800k. The required down payment is 40%. He can borrow the rest 60% from the bank. The loan must be paid back in 25 years with annual payments. James currently has $400k in savings. Next annual salary is $60k to be paid 1 year from today. The salary will grow at annual rate of 2%. James consumes 50% of annual salary. The bank charges annual interest rate 6%. Should the bank extend the loan to James?
    - Total amount of loan: $800k*60%=$480k.
    - Next salary for loan payment: $60k*50%=$30k.
    - $PV_0 = \frac{30000}{6\%-2\%}\left(1 - \left(\frac{1+6\%}{1+2\%}\right)^{25}\right) = \$463.3k < \$480k$. So the bank should reject James' loan based on the income alone
    - However, since James has $400k savings and the downpayment is $320k, the bank may still consider extending the loan.
  - Delayed problem can be solved in a similar way as in the previous example

Compounding frequency

- Compounding frequency is the frequency at which interest is accrued or re-invested within one year
- It is the frequency at which interest is compounded on interest
- If we invest $100 for one year at 12% per year, compounded quarterly, how much do we have at the end of three years?
  - 12% is the annual percentage rate (APR)
    - Nominal rate
    - It is the rate used to calculate the annual interest (payoff), but how frequent interest is distributed and how much we receive in each distribution depends on the frequency of compounding
  - The normal annual pay-off is 12%, but it is distributed 4 times a year (quarterly compounding), each distribution gives 12%/4=3% of payoff in each quarter
  - The 3% pay-off is immediately re-invested once distributed
  - The total number of compounding periods over the 3 years is 12
  - $PV = 100\left(1 + \frac{12\%}{4}\right)^{4*3}$.
    - Effective rate per period: the interest rate for each period when interests or pay-

offs are paid and reinvested.

- Compounding saving m times per year for T will have value in T years:
  - $PV = C_0 \times \left(1 + \frac{r}{m}\right)^{mT}$.
  - C_0 is the initial saving, r is the annual percentage rate, m is compounding frequency.
  - Effective rate per period: $i = \frac{r}{m}$.
  - Total number of payments/periods: $n = mT$.
- To have $\$C_T$ in T years, with APR of r, compounded m times a year, we need to save $PV = C_T\left(1 + \frac{r}{m}\right)^{-mT}$.
- Effective Annual Rate (EAR): $EAR = \left(1 + \frac{r}{m}\right)^{m} - 1$.
  - Saving at APR of r per year, compounded m times a year is equivalent to saving at EAR per year, compounded annually.
  - EAR is the annual rate of return that gives the same amount of saving wealth at the end of the saving period.
  - EAR allows for the comparison of APRs compounded at different frequencies
- E.g. Suppose Bank A has $r = 12\%$ compounded quarterly and Bank B has $r = 11\%$ compounded monthly.
  - $i_A = \frac{12\%}{4} = 3\%$, $EAR_A = (1 + 3\%)^4 - 1 = 12.55\%$.
  - $i_B = \frac{11\%}{12} = 0.91\%$, $EAR_B = \left(1 + \frac{11\%}{12}\right)^{12} - 1 = 11.57\%$.

Canadian Mortgage

- Debt payments are flat
- Canadian banks quote the annual interest rate compounded semi-annually for mortgages
- Interest is compounded monthly for monthly mortgage payments (debt payments)
- The terms of the mortgage are usually negotiated during the term of the mortgage
  - The interest of a 25-year mortgage can be negotiated 5 years after the initiation of the mortgage
- Fixed-rate and variable rate
  - Fixed-rate: the mortgage interest rate stays constant over the 5 year term
  - Variable-rate: the mortgage rate adjusts according to the benchmark interest rate in Canada
- e.g. 25 year \$300k fixed-rate mortgage at 2.9% per year compounded semi-annually. What is the monthly payment.
  - The quoted rate (APR) is compounded semi-annually, but monthly payments occur monthly. Need to compute the effective rate per month (two arrangements must have the same EAR)
  - $EAR_{bank} = \left(1 + \frac{2.9\%}{2}\right)^2 - 1 = EAR_{my\ side} = (1 + i)^{12} - 1$, so $i = 0.2402\%$.
  - Assume we pay \$C each month. The total number of periods for monthly payment is 300 =12*25.
  - $\$300k = PV = \frac{C}{i}\left(1 - \frac{1}{(1+i)^n}\right)$, so $C = \$1{,}404.35$.

Cash flows at different frequencies

- $r = 10\%$ interest rate (actual rate of return) per year.
- Two investment projects, (A) \$100 per year for 3 years; (B) \$35 per quarter for 2 years.
- $PV_A = \frac{100}{10\%}\left(1 - \frac{1}{1.1^3}\right) = \$248.69$.
- For B, need to find effective r_Q that is equivalent to EAR of $r = 10\%$, $\left(1 + r_Q\right)^4 - 1 = 10\%$, $r_Q = 2.411\%$.
- $PV_B = \frac{35}{2.411\%}\left(1 - \frac{1}{1.02411^8}\right) = \$251.91$.

Continuous compounding

- Given APR, as compounding frequency increases, EAR will increase.
- If compounding frequency is infinite (continuous compounding), then $FV = C_0 e^{rT}$.

- $EAR = \left(1 + \frac{r}{m}\right)^m - 1$ is increasing when m increases, but it is capped at $e^r - 1$.

Annuity due

- Annuity due is a type of annuity that pays cash flows at the beginning of each period
- $PV_{-1} = \frac{C}{r}\left(1 - (1+r)^{-T}\right), PV_0 = (1+r)PV_{-1}$.

Net present value (NPV)

- NPV=PV of cash inflows - PV of cash outflows = PV of investment income - PV of cost of investment
- To compute PV, we use interest rate in the financial market or rate of return on similar investment.
- The NPV rule of investment: we should accept investment opportunity as long as NPV≥ 0 (basic principle of investment).
- Basic principle implies the NPV rule.
- e.g. suppose we have \$C to invest for 1 year The financial market offers an interest rate of r_F per year. An investment opportunity offers a payment of \$C(1+r_i) in 1 year, but requries initial investment of \$C.
    - By Basic principle, accept if $r_i \geq r_F$, we show that $r_i \geq r_F$ implies NPV≥ 0.
    - Now: -\$C, an outflow of \$C.
    - 1 year later: $+\$C(1+r_i)$ cash inflow (investment income).
    - NPV of the investment using rate from the financial market, $NPV = -C + C\frac{1+r_i}{1+r_F} = C\left(\frac{1+r_i}{1+r_F} - 1\right)$.
    - If $r_i \geq r_F$, then $NPV \geq 0$.

Bond Valuation

May 29, 2024 12:00 PM

Bond: a bond is a legally binding agreement between a borrower (bond issuer) and a lender (bondholder)

- Specifies the principal amount of the loan: lump-sum payment due at the end of term of borrowing
- Specifies the timing and size of the cash flows:
  - Periodical coupon payments; lump-sum principal payment
  - Fixed amount (fixed rate bond)
  - Variable: inflation-linked bonds
- Par (Face) Value
  - The stated (principal) amount due on maturity date
  - In bond quoting, the price is quoted per $100 of actual Par value
  - Different bonds can have different Par value. We can multiply the corresponding ratio between Par Values and 100 to arrive at the actual price.
- Maturity date:
  - The date on which the principal amount of the debt must be repaid
  - Time to maturity: the remaining time until the maturity date, usually measured in years
- Yield to maturity (YTM)
  - The rate of return per annum (expressed as an APR) on a bond if it is held till maturity
- Coupon payment
  - Periodical interest payment
- Coupon rate
  - The interest rate per Par (Face) Value to calculate the coupon payment
- Premium:
  - If bond price > par, the difference price - bar is the premium
  - The bond is called a premium bond, the bond sells at a premium
- Discount:
  - If bond price < par, the difference bar - price is the discount
  - The bond is called a discount bond, the bond sells at a discount
- If bond price is the same as Par Value, the bond sells at Par
- Holding period return (HPR)
  - The rate of return earned for the period of holding of the bond (not to maturity)
  - HPR $= \frac{ending\ price\ - beginning\ price + coupon\ payments}{beginning\ price}$.
    - Beginning price is the price we buy the bond at (costs).
    - Ending price is the price we sell the bond at (receipts).
    - HPR may not be annual rate
- Current yield
  - The ratio of annual coupon payments to bond price
  - Current yield $= \frac{total\ annual\ coupon}{bond\ price}$.
  - Current yield is an annual rate

E.g.:

- Coupon rate: 6.375
- Maturity Date: Dec 31, 2030
- Bid $: 107.05
- Yield: 5%
- Government of Canada bond with a Par (Face) Value of $100,000 is quoted at $107.05 and we want to buy one unit of this bond. What is the price we have to pay?
- The ratio of Par value to $100 is 1000=$100k/$100
- The bond is currently selling for $107.05. We need to pay $107.05 per $100 of such bond
- The actual price we have to pay on one bond is $107,050=1000*$107.05

E.g.

- Face value: $5k
- Maturity date: Aug 15, 1986
- Annual coupon: $400
- Coupon rate: 8%=400/5k
- If this bond was priced at $102 on Jan 15, 1985, was it sold at par?
  - Actual price of this bond: $5,100=5000/100*$102

Bond valuation

- Bond value is determined by the present value of the coupon (if any) and face value payments

Pure discount (zero-coupon) bond

- Information needed:
  - Time to maturity T = maturity date - today's date, expressed as the number of periods that YTM rate (i) applies to
  - Face value F
  - Discount (YTM) rate I.
- Price: $PV = \frac{F}{(1+i)^T}$.
- e.g. value of a 30-year zero-coupon bond with a face value of $1,000 and a YTM of 6%.
  - $PV = \frac{F}{(1+i)^T} = \frac{1000}{(1+0.06)^{30}} = 174.11.$
  - Ratio is $1000/100=10, so the quoted price is 174.11/10=17.411.

Level-coupon bonds

- Information needed
  - Coupon payment dates and term to maturity (T)
    - T is the number of coupon payments (periods) to maturity date
  - Coupon payment C per period and face value (F)
    - Coupon payment is fixed, and thus coupon rate is fixed
  - Discount rate: I per coupon payment period
- Value of a level-coupon bond = PV of coupon payment annuity + PV of face value
  - $PV = \frac{C}{i}\left[1 - \frac{1}{(1+i)^T}\right] + \frac{F}{(1+i)^T}$.
- e.g. find the PV (at 2011-01-01) of a 6-3/8 coupon (coupon rate= $\left(6 + \frac{3}{8}\right) = 6.375\%$) bond with semi-annual payments, and a maturity date of 2016-12-31. The YTM is 5%.
  - Face value: $1000.
  - Coupon payment at 06-30 and 12-31
  - Annual coupon payment: $6.375\% * 1000 = \$63.75$, but the actual semi-annual coupon payment is 0.5*63.75=$31.875.
  - YTM=5%, then i =5%/2=2.5% (effective rate per 6 month).
  - $PV = \frac{31.875}{2.5\%}\left(1 - \frac{1}{(1+2.5\%)^{12}}\right) + \frac{1000}{(1+2.5\%)^{12}} = \$1070.52.$

Bond price between coupon dates

- Clean price: the price quoted on the bond market
  - The value of the bond exactly one period before the next coupon payment
  - Does not include the part of interest that the seller is entitled to (the accrued interest)
- Dirty price: the full/invoice price
  - Includes the accrued interest to the seller
  - The price that a buyer actually pays
- Clean price + accrued interest = dirty price

Yield to maturity (YTM)

- The annual percentage rate (APR) of return of a bond if you purchase and hold it until maturity
- The compounding frequency of YTM is the same as the frequency of coupon payment
- Mathematically, YTM is the APR that equates the PV of the remaining cash flows of a bond to

its current sale price

- $PV = \frac{C}{YTM/m}\left(1 - \frac{1}{(1+YTM/m)^{mT}}\right) + \frac{F}{(1+YTM/m)^{mT}}$.

- Given bond price, the YTM is the discount rate that solves the above equation
- YTM is also the rate of return that investors require to earn when holding bond to maturity
- e.g. a bond pays semi-annual coupon at a coupon rate of 10%. It has 2.5 year to maturity. The current quoted price is $102.05. What is YTM
  - Coupon payment: C=$5=($100 * 10%)/2
  - Remaining periods: n=5=2*2.5
  - Quoted price: $102.05, face value=Par=$100
  - $102.05 = \frac{5}{YTM/2}\left(1 - \frac{1}{\left(1+\frac{YTM}{2}\right)^5}\right) + \frac{100}{\left(1+\frac{YTM}{2}\right)^5}$.
  - This gives YTM=8.863%.

Bond price and YTM

- Bond price and YTM move in opposite directions
  - Market interest rates move in the same direction as YTM
  - Bond price and market interest rates move in opposite directions
- If coupon rate = YTM, quoted price = par value
- If coupon rate > YTM, quoted price > par value (premium bond)
- If coupon rate < YTM, quoted price < par value (discount bond)
- Higher YTM, lower bond price
- A bond with longer term to maturity has a higher relative price change than one with shorter maturity when interest rates and YTM change. All other features are identical
  - Par value $100, A has 30 years to maturity, B has 5 years to maturity
  - $P_A = \frac{100}{(1+YTM)^{30}}, P_B = \frac{100}{(1+YTM)^5}$.
  - Note that YTM is a type of return, will be affected by risk-free rate, inflation, etc.
  - If YTM increases, price of A will have a larger fall than B in value.
- A lower coupon bond has a higher relative price change than a higher coupon bond when interest rates and YTM change. All other features are identical
  - Suppose Par value $100, A has 15% coupon rate, and B has 5% coupon rate
  - B will be more sensitive to interest rate changes. Because more weight is on the par value, and it is deeply affected by the interest rate.
  - Higher coupon rate means that more coupon values are discounted in a shorter period. Less sensitive to interest rates.
  - If coupon rate is lower, more bond price comes from PV of Face value which is more deeply discounted by YTM as it is in the distant future.
- Long maturity bond is more volatile w.r.t. changes in the discount rate
  - Bond price with longer maturity decreases much faster w.r.t. YTM than bond price of shorter maturity

Holding period return

- Gain (loss) from holding a bond for resale
  - Increase (decrease) in bond price
  - Coupon payments received during the holding period
- $r_{hold} = \frac{P_{sell} - P_{purchase} + coupon\ payments}{P_{purchase}}$.
  - $\frac{P_{sell} - P_{purchase}}{P_{purchase}}$ is the capital gain yield, and $\frac{coupon\ payments}{P_{purchase}}$ is the current yield.

Yield curve

- Even for the same type of bond, YTM can be different with different terms to maturity
- The yield curve is a plot of YTM against the term to maturity of the same type of bond
  - Similar bonds should have identical default risk
- The yield curve can have different shapes
  - Future interest rate may change
  - People may have different preferences over time to maturity

Spot and forward rates

- Spot rate
  - The prevailing interest rates (YTM) implied by the bond price in the current market
  - Usually, we use zero coupon bond to construct spot rate
  - Spot rates can have different terms as bonds with different time to maturities are used to construct them
- Forward rate
  - Rate determined in a forward interest rate contract
  - The future short-term interest rates implied by the spot rates
    - In an ideal world (no risk, no preference over investment horizon), forward rates is the future short-term interest rates
    - But in general, forward rates are not the same as future short-term interest rates
- e.g. at date 0, zero coupon Canada bonds with different maturities are traded, Par Value $1000
  - Prices (spot): 1-year: %980, 2-year: $930, 3-year: $890.
  - 1-year spot rate (S_1) : 2.04%=1000/980-1.
  - 2-year spot rate (S_2): 3.695%=$\left(\frac{1000}{930}\right)^{1/2} - 1$.
  - 3-year spot rate (S_3): 3.96%=$\left(\frac{1000}{890}\right)^{1/3} - 1$.
  - Suppose I want to save money for 2 years, but I don't want the 2-year bond.
    - I can buy 1 year bond with interest rate 2.04%. Then lock in constant rate for both year 1 and year 2, gives a forward rate of f_2. In total, profit interest rate is (1+ 2.04%)(1+f_2).
    - Or I can buy 2-year bond with interest rate $(1 + 3.695\%)^2$.
    - Equating these two options gives a forward rate at year 1 (f_2): 5.376%.
  - Similarly forward rate at year 2 (f_3): 4.494%, by $(1 + 3.695\%)^2(1 + f_3) = (1 + 3.96\%)^3$.
- In general, $f_n = \frac{(1+S_n)^n}{(1+S_{n-1})^{n-1}} - 1$.
  - S_n is the n-year(n-term) spot rate.
  - f_n is the forward rate from period $n - 1$ to n.
- Forward rates and future short-term rates.
  - In an ideal world, forward rates is the future short-term rates
  - Otherwise, arbitrage opportunity will arise.
- e.g. two zero coupon bonds (zero): same risk, face value (FV)
  - Long term (n-year) zero: $P_0^L = \frac{FV}{(1+S_n)^n}$.
  - Short term (n-1-year) zero: $P_0^S = \frac{FV}{(1+S_{n-1})^{n-1}}$.
  - If $f_n > r_n$, it implies the forward rate is too high, then either S_n too high and P_0^L too low, or S_{n-1} too low and P_0^S too high.
  - Consider the following arbitrage strategy: short 1 unit of short-term zero and long $\frac{P_0^S}{P_0^L}$ units of long-term zero, the cash flow from time 0 to time n will be
  -

	Time 0	Time n-1	Time n
In flow	Receipts from short P_0^S=FV/$\left(1 + S_{n-1}\right)^{n-1}$.	Borrow FV to close the short position FV	Receipt from long term zero: (P_0^S/p_0^L)FV=$\frac{1+S_n}{1+S_{n-1}}FV$
Out flow	Pay for long $-P_0^S$.	Pay to close the short position -FV.	Pay previous debt -(1+r_n) FV.
Total	0	0	$FV(f_n - r_n) > 0$.

Expectation Hypothesis

- In real world, no one knows the future exactly, previous arbitrage mechanism is not possible, therefore future short-term interest rate can deviate from forward rate

- But the yield curve reflects market consensus about the expected future interest rates $f_n = E(r_n)$.
- Shape of the yield curve reflects the movement of expected future short-term interest rates
- Important assumption: short-term and long-term bonds are perfect substitutes

Liquidity Preference Hypothesis

- People have different preferences over investment horizon
    - For short-horizon investors, they may prefer short-term bonds as they give cash payment early (more liquid). Require more discounts on longer-bonds for these investors to buy them $f_n > E(r_n)$.
    - For long-horizon investors, $f_n < E(r_n)$.
- In general, $f_n = E(r_n)$ +liquidity premium.
    - The liquidity premium can be positive or negative, depending on investors' preferences.
    - It is more likely to be positive

Shapes of yield curve

- Steeply rising: future short-term interest rates will be rising and expect economic expansion
- Modestly rising: cannot predict the future economy
- Declining: future economy will slow down (assuming a constant liquidity premium over time)

Stock Valuation

June 10, 2024 10:33 AM

Valuation of common stock

- The value of any asset is the present value of its expected future cash flows
- Stock ownership produces cash flows from
  - Dividends: direct cash flows from dividend payments
  - Future sale proceeds (secondary market)
    - Results in capital gain/loss
    - Capital gain/loss = $P_{sell} - P_{buy}$.

Pure dividends vs. future sales

- If hold the stock forever:
  - $P_0 = \sum_{t=1}^{\infty} PV_0 \ of \ Div_t$.
- If sell the stock at time k.
  - $P_k = PV_k \ of \ Div_{k+1} + PV_k \ of \ Div_{k+2} + \cdots$.
  - $PV_0 \ of \ P_k = PV_0 \ of \ Div_{k+1} + PV_0 \ of \ Div_{k+2} + \cdots$.
  - $P_0 = \sum_{t=1}^{\infty} PV_0 \ of \ Div_t$.

Dividend-discount model

- Evaluating common stocks based on future sale prices and dividends are equivalent
  - The intrinsic value of a common stock arises from the future dividend cash flows
  - If a stock doesn't pay dividend
    - It is expected that dividends will ultimately be paid at some point of time
    - Value the growth opportunities
  - In general, the amount of dividend is uncertain
- The dividend discount model
  - A simplified model for dividend payout
  - Discount the dividend cash flows according to model assumptions
  - Zero growth, constant growth, differential growth

Zero growth

- Assume that dividends will remain at the same level forever
  - $Div_1 = Div_2 = \cdots = Div$.
- Since future cash flows are consistent, the value of a zero growth stock is the present value of a perpetuity
  - $P_0 = \frac{Div_1}{1+r} + \frac{Div_2}{(1+r)^2} + \cdots = \frac{Div}{r}$.
  - Where r is the relevant discount rate per dividend paying period.
  - r is similar to YTM in bond valuation: the required return on the stock.
  - It is affected by the rate of return on similar firms, riskiness of the firm and the preferences of the investors.

Constant growth

- Assume that dividends will grow at a constant rate, g forever.
- $Div_{t+1} = Div_t(1+g)$.
- Since the future cash flows grow at a constant rate forever, the value of a constant growth stock is the present value of a growing perpetuity: $P_0 = \frac{Div_1}{r-g}$.

Differential growth

- Assume that dividends grow at rate g_1 for N years, and grow at rate g_2 thereafter.
  - $Div_N = Div_{N-1}(1+g_1) = Div_1(1+g_1)^{N-1}$.
  - $Div_{N+1} = Div_N(1+g_2) = Div_1(1+g_1)^{N-1}(1+g_2)$.

- The value is the sum of an N-year annuity growing at rate g_1 plus the discounted value of a perpetuity growing at rate g_2 that starts in year $N+1$.
    - $P_{0A} = \frac{Div_1}{r-g_1}\left(1-\left(\frac{1+g_1}{1+r}\right)^N\right)$.
    - $P_{0B} = \frac{\left(\frac{Div_{N+1}}{r-g_2}\right)}{(1+r)^N} = \frac{\frac{Div_1(1+g_1)^{N-1}(1+g_2)}{r-g_2}}{(1+r)^N}$.
    - Price at date 0: $P_0 = P_{0A} + P_{0B}$.

Terminologies
- Earnings: net income
- Earnings per share (EPS): earnings/total common shares
- (dividend) payout ratio: total dividend / earnings
    - Alternatively: dividend per share/EPS
- Retained earnings: part of net income that is not distributed as dividends
- Retention ratio: retained earnings this year/earnings this year
    - Alternatively: 1-divident payout ratio
- Return on equity (ROE): net income/total equity
- Dividend yield: dividend per share/price

Estimating parameters in the model:
- Growth rate g:
    - The firm will experience earnings growth if its net investment (total investment-depreciation) is positive
    - To grow, the firm must retain some of its earnings:
    - Earnings next year = earnings this year + retained earnings this year * return on retained earnings.
    - Return on retained earnings can be approximated by returns on equity
    - Divide everything by earnings this year, and LHS=$1+g$, (retained earnings this year)/(earnings this year)=retention ratio.
    - Therefore, g =retention ratio * return on retained earnings.
    - The return on retained earnings can be estimated using the firm's historical return on equity (ROE)
    - e.g. A company just reported earnings of $1.6 million and it plans to retain 28% of its earnings. If the historical ROE was 12%, what is the expected growth rate for its earnings?
        - $g = 0.28 * 0.12$.
- Discount rate r:
    - It reflects investors' required rate of return on the stock holding
        - Specific discount rates could vary across different investors
        - What's relevant is the overall discount rate prevailing in the market that determines the stock price
    - To estimate r, we utilize the avilable info $r = \frac{Div_1}{P_0} + g$ fo constant growth case.
    - The discount rate can be broken into two parts.
        - Dividend yield: $\frac{Div}{P_0}$.
        - The growth rate: g.

The NPVGO (net present value of growth opportunity) model
- To price a stock that pays no dividends
- Growth opportunities are opportunities to invest in positive NPV projects
- The value of a firm can be conceptualized as the sum of
    - The value of a firm that pays out 100% of its earnings as dividends (cash cow)
    - The net present value of the growth opportunities per share
- $P = \frac{EPS}{r} + NPVGO$.
- e.g. A company currently has EPS of $1.5, with 1 million shares outstanding. The annual dividend is equal to the EPS. This is expected to occur in perpetuity if the company makes no

new investments. The company has decided to spend $1.5 million on a new project that will increase the earning by $250k. The company's discount rate is 12%

- NPV of the new project: -$1.5 million + $250k/12%=$583,333.
- NPVGO= $583k/1.12=$520.833k
- NPVO per share=$520k/1million=$0.52.
- Price of share: $P = \frac{\$1.5}{0.12} + 0.52 = \$13.02.$

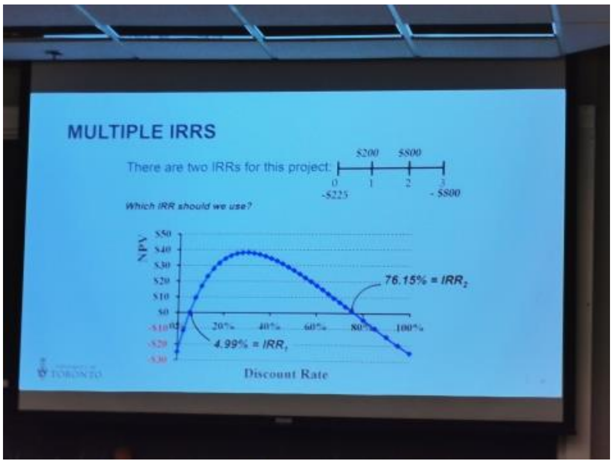
Note: the dividend discount model and the NPVGO model should give the same price of a share

Price-earnings (PE) ratio

- PE ratio=price per share/EPS.
  - Also known as PE multiple
  - The national post sues last 4 quarters' earnings to estimate EPS
- Firms whose shares are in fashion sell at high multiples (growth stocks)
- Firms whose shares are out of favor sell at low multiples (value stocks)
- Relating the PE ratio to the NPVGO approach
  - PE ratio $= \frac{1}{r} + \frac{NPVGO}{EPS}.$
  - PE multiple is negatively related to r, as r increases, the PE multiple declines.
  - PE multiple is positively related to g, as g increases, the PE multiple increases.
  - Accounting methods to determine EPS, will impact the PE ratio for that company's share
  - In general, companies with conservative accounting practices will have higher PE multiples.

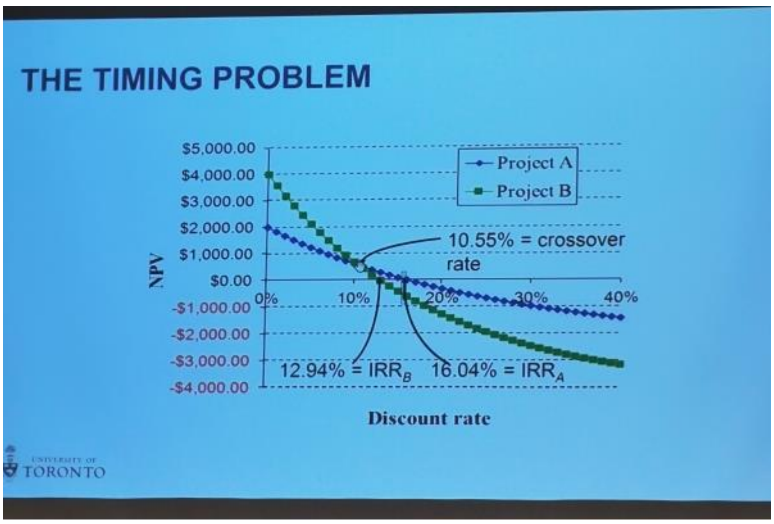
Common valuation rules

- Net present value (NPV) rule
  - NPV: PV of cash inflows - PV of cash outflows
  - Estimating NPV:
    - Estimate future cash flows
    - Estimate discount rate
    - Estimate initial costs
  - Minimum acceptance criteria: accept if NPV>0
  - Ranking criteria: choose the highest NPV
  - Reinvestment assumption
    - All cash flows can be reinvested at the discount rate
  - Discount rate: the required rate of return
    - The rate of return a firm can earn on similar projects. Also known as the investment rate
    - Riskiness of the investment
    - Returns on alternative investment options
- The internal rate of return (IRR) rule
  - IRR: the discount rate that sets NPV to zero (similar to YTM in bond valuation).
  - downward-sloping NPV-discount rate profile:
    - Minimum acceptance criteria: accept if IRR> required rate of return (NPV positive)
    - Ranking criteria: Highest IRR
  - Upward-sloping NPV-discount rate:
    - Minimum acceptance criteria: accept if IRR< required rate of return (NPV positive)
    - Ranking criteria: lowest IRR
    - Usually occurs when there are significant negative cash flows in the distant future
  - Reinvestment assumption:
    - All future cash flows assumed reinvested at the IRR
  - Disadvantages
    - Does not distinguish between investing and borrowing
    - IRR may not exist or there may be multiple IRR
    - Problems with mutually exclusive investments
  - Advantages
    - Easy to understand and communicate

        - Summarize project using one rate
- Multiple IRRs
    - 

    - Should accept if the IRR leads to positive NPV
- Modified IRR
    - Calculate the present value of all cash outflows using the borrowing rate
    - Calculate the future value (at the end of the project) of all cash inflows using the investing rate
    - Find the rate of return that equates these values
    - Benefits: single answer and specific rates for borrowing and reinvestment
    - e.g. suppose the required return (investment rate) is 4% and the borrowing rate is also 4%, calculate MIRR
        - FV of cash inflows (at time 3, income of investment): FV=1048.32=$200 * 1.04^2 + 800 * 1.04$.
        - PV of cash outflows (at time 0, cost of investment): PV=936.19$= 225 + 800/1.04^3$.
        - $PV(1 + MIRR)^3 = FV$.
        - This gives MIRR=3.84%<4%, should reject

Mutually exclusive project
- Only one of several potential projects can be chosen
- Scale problem and timing problem
- Rank all alternatives and select the best one



- Always choose the one with highest NPV
- When discount rate < cross over rate, choose project B, otherwise, choose project A

Independent project
- Accepting or rejecting one project does not affect the decision of the other projects
- Accept ones above a minimum acceptance criteria

Risk and Return

June 12, 2024 11:31 AM

Holding Period Return (HPR)

- Bond: $\frac{coupons+change\ in\ bond\ price}{purchase\ price}$.
- Stock: $\frac{dividends+change\ in\ market\ value}{purchase\ price}$.
- Change in bond price/market value are the capital gain/loss.
- e.g. (stock) suppose you bought 100 shares of BCE two years ago at $25 per share. Over the last year, you received 20 cents per share of dividends. The stock now sells for $30 per share.
  - The HPR over the 2 years is $\frac{\$0.2*100+(\$30-\$25)(100}{\$25*100}=\frac{\$520}{\$2500}=20.8\%$.
  - The effective annual return is $(1+EAR)^2=1+20.8\%$, and it gives EAR=9.91%.

Average return

- Arithmetic average annual return over n years: $r_A=\frac{1}{n}\sum_{i=1}^{n} r_i$.
  - Return earned in an average period over multiple periods
  - Expected return for a regular period over multiple periods
  - Also referred to as the mean return
- Geometric average annual return over n years: $r_g=\left[\prod_{i=1}^{n}(1+r_i)\right]^{\frac{1}{n}}-1$.
  - Average compounded return per period over multiple periods

Return statistics

- The history of capital market returns can be summarized by describing the
  - Average return (approximation of performance): $r_A=\frac{1}{n}\sum_{i=1}^{n} r_i$.
  - Sample variance of the returns: $\sigma^2=\frac{1}{N-1}\sum_{i=1}^{n}(r_i-r_A)^2$.
  - Standard deviation of the returns (risk measurement/volatility): σ.

Average stock returns and risk-free returns

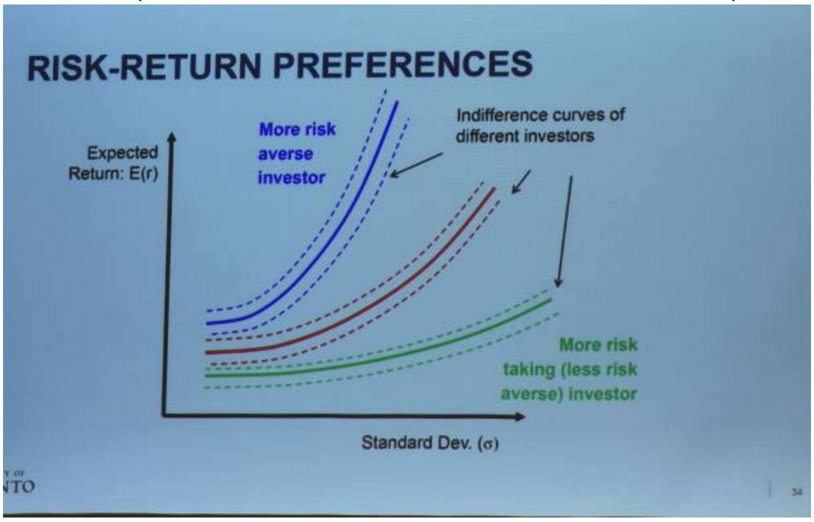
- The **risk premium** is the additional return (over and above the risk-free rate) resulting from bearing risk
  - $r_{risk}-r_{rf}$.
  - It is a measure of riskiness
- One of the most significant observations of stock and bond market data is this long-run excess of security return over the risk-free return
  - The average excess return (risk premium) from Canadian common stocks for 1957-2009 is 4.45%=10.7%-6.35%
  - The average excess return from Canadian long-term bonds for 1957-2009 is 2.17%= 8.52%-6.35%
- Rate of return on T-bills is essentially risk-free
- Investing in stocks is risky, but there are compensations
- The difference between the return on T-bills and stocks is the risk premium for investing in stocks

Risk and return

- The investors care about the risk and return when choosing financial assets
- Risk: usually measured by the standard deviation of the returns of a security
- Return: realized in future when the asset is sold
  - Uncertain, needs to estimate
- Expected return $E(r)$:
  - If we do not know the distribution of returns, then $E(r)=\frac{1}{n}\sum_{i=1}^{n} r_i$.
  - If we know the probability when r_i occurs, then $E(r)=\sum_{i=1}^{n} P_{r_i} r_i$.

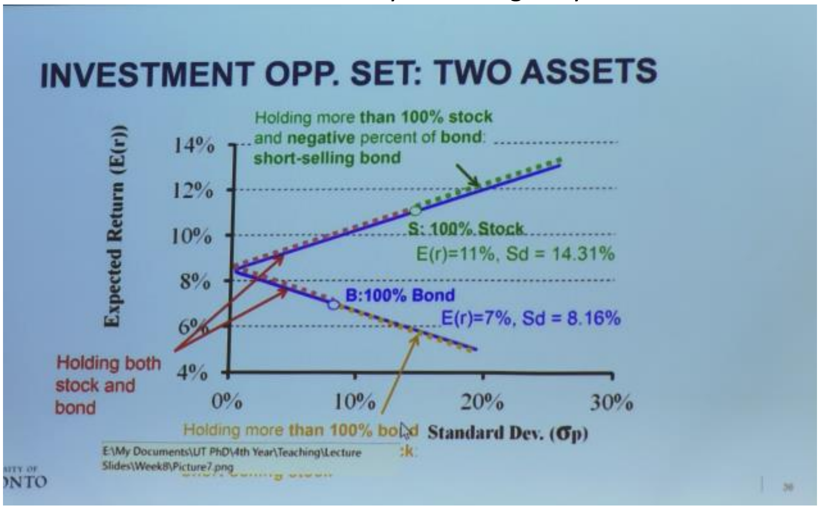
- Variance of returns
  - Without distribution, $\sigma^2 = \frac{1}{N-1}\sum_{i=1}^{n}\left(r_i - E(r)\right)^2$.
  - With distribution, $\sigma^2 = \sum_{i=1}^{n} P_{r_i}\left(r_i - E(r)\right)^2$.

Portfolio risk and returns
- A portfolio is a combination of different assets or securities
- Expected return of a portfolio depends on
  - Expected return of the individual assets
  - Weight (proportion) of each asset
    - This weight is always value weighted, measured by the dollar amount invested in an asset relative to the total dollar amount invested in the whole portfolio
- Suppose there are two assets in a portfolio with weights w_1, and w_2.
  - Expected return: $E(r_P) = w_1E(r_1) + w_2E(r_2)$.
  - Variance of the return: $Var_r = \sigma_r^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2Cov(r_1, r_2)$.
    - $Cov(r_1, r_2) = \rho_{1,2}\sigma_1\sigma_2$.
    - $\rho_{1,2}$ is the correlation coefficient
- Standard deviation of the return (risk) of a portfolio depends on
  - Standard deviation of individual assets
  - Weight of each asset
  - Correlation of returns among different assets
  - Covariance
    - With distribution: $Cov(r_1, r_2) = E\left(\left(r_1 - E(r_1)\right)\left(r_2 - E(r_2)\right)\right)$.
    - Without distribution:
      - Sample: $Cov(r_1, r_2) = \frac{1}{n-1}\sum_{i=1}^{n}(r_{1i} - \overline{r_1})(r_{2i} - \overline{r_2})$.
      - Population: $Cov(r_1, r_2) = \frac{1}{n}\sum_{i=1}^{n}(r_{1i} - \overline{r_1})(r_{2i} - \overline{r_2})$.

Risk-return preferences
- Investors have tastes over risks and expected return
  - Given a level of expected return, investor would like lower standard deviation (volatility) of the return (lower risk)
  - Given a high level of volatility (higher risk), investor require higher expected return to compensate for risk
  - In a space of risk and return, the indifference curve is upward sloping
- 

Investment opportunity set: two assets
- The investment opportunity set is a combination of expected return and volatility of the portfolio that can be achieved by changing the weights of assets
- The efficient frontier
  - The portion of the opportunity set which satisfy the condition that no other portfolio exists with a higher expected return but with the same risk

- The upper portion of the investment opportunity set
- The best we can achieve by allocating risky assets.

- 

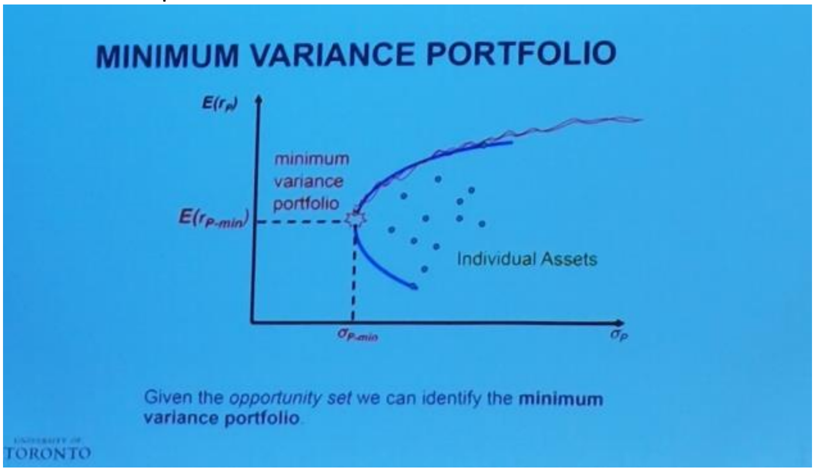
- The optimal allocation is the intersection of indifferene curve and efficient frontier
  - Shape of the efficient frontier depends on the correlation coefficient
  - The smaller the correlation, the greater the risk reduction potential
    - If $\rho = -1$, complete risk reduction is possible.
    - If $\rho = 1$ (perfectly correlated, no risk reduction is possible.

CAPM

June 17, 2024 11:25 AM

Consider a world with many risky assets, we can still identify the opportunity set of risk-return combinations of various portfolios.

Minimum variance portfolio

- 

- Min variance portfolio is obtained by solving the problem
  - $\min \sigma_p^2$ s.t. $w_1 + w_2 = 1$.
  - Minimum is the min of $(w_1\sigma_1)^2 + ((1-w_1)\sigma_2)^2 + 2w_1(1-w_1)\sigma_1\sigma_2\rho_{1,2}$.
  - $w_1 = \frac{\sigma_2^2 - cov(r_1,r_2)}{\sigma_1^2+\sigma_2^2-2cov(r_1,r_2)}$.
- The section of the opportunity set above the minimum variance portfolio is the efficient frontier.

Riskless borrowing and lending

- The capital allocation line (CAL) shows all feasible risk-return combinations of a risky and a risk-free asset.
  - The best we can achieve when we allocate both risky and risk-free assets
- CAL: $E(r) = r_f + \frac{E(r_P)-r_f}{\sigma_P}\sigma$.
- At $\sigma = 0$, $w_p = 0, w_f = 1$.
- At intersection with the efficient frontier, $w_p = 1, w_f = 0$.
- Along the extension, $w_p > 1$, $w_f < 0$.
  - Borrowing money at risk-free rate
- Tangency portfolio: The best achievable CAL is the CAL that is tangential to the efficient frontier. The tangency point is the tangency portfolio.

Sharpe ratio

- Sharpe ratio is the slope of the CAL: $\frac{E(r_P)-r_f}{\sigma_P}$.
  - It is a form of risk-adjusted rate of return
- Investors would like to choose the capital allocation line with the steepest slope (maximize the Sharpe ratio)
- The steepest CAL becomes the Capital Market Line (CML).

Capital Market Line

- $E(r_P) = r_f + \frac{E(r_M)-r_f}{\sigma_M}\sigma_P$.
  - r_M is the market return, σ_M is the variance of market portfolio.
- Risk premium of market portfolio $E(r_M) - r_f$ is affected by
  - Average risk preferences of the investors. If investors are more risk averse, higher risk

    - premium.
    - The variance of the market portfolio. Higher variance (risk) means market portfolio is riskier, investors require higher return to compensate, higher risk premium.

Separation property

- Portfolio choice can be separated into two tasks
    - Determine the optimal risky portfolio
    - Selecting a point on the CML

The optimal risky portfolio depends on the risk-free rate as well as the risky assets

Capital Asset Pricing Model (CAPM)

- Linear relationship between the expected return of the individual security and that of the market portfolio
    - At market equilibrium, each investor holds the market portfolio composed of each individual asset
- The expected return of security or portfolio i.
    - $E(R_i) = R_f + \beta_i\left(E(R_M) - R_f\right)$.
    - This applies to individual securities held within well-diversified portfolios (cross-sectional)
- The expected return of the market portfolio.
    - $E(R_i) = R_f +$ market risk premium.
- The relationship between asset return, risk free rate, and market portfolio return is called the Capital Asset Pricing Model (CAPM)
    - Expected return on a security = risk-free rate + beta of the security * market risk premium (risk premium of asset i)
    - Beta measures the co-movement of the return of individual security with that of the market portfolio (sensitivity of a security to the market, risk)
        - $\beta_i = \frac{cov(r_i, r_m)}{\sigma_m^2}$.
        - The risk premium of the individual security is proportional to the risk premium of the market portfolio.
        - If $\beta_i = 0$, then the expected return is r_f.
        - If $\beta_i = 1$, then $E(R_i) = E(R_M)$.
- We can estimate β with regression. The fitted line is called the characteristic line.
    - Regress security returns to market return (e.g. SP500 return).
- The graphical representation of CAPM is the Security Market Line (SML) (with x-axis being β_i).
    - It measures the relationship between the expected return of asset i and β_i.
    - CAPM predicts a straight SML
    - The slope of the SML is the risk premium, and it should always be positive.
- Assumptions of CAPM
    - Investors have homogeneous expectations (same belief and expectation about security performance)
    - Investors are price takers (transactions cannot influence prices)
    - Investors are rational mean-variance optimizers and face single period investment horizon
    - Investors are limited to traded financial assets
    - Investors face no taxes or transaction costs
    - Investors can costlessly access information available to all

Systematic and unsystematic risk

- Systematic (undiversifiable) risk
    - Affects a large number of assets
    - May affect the entire market
    - e.g. uncertainty about general economic conditions, GNP, interest rates, inflation
- Unsystematic (diversifiable) risk

- Specifically affects a single asset or small groups of assets
- Unsystematic risk can be diversified away by holding a large variety of assets (securities)
- e.g. announcements specific to a company, such as a gold mining company striking gold

- The actual return an asset can be written as $r_i = E(r_i) + u_i$.
- The risk can be broken down into two components: $u_i = m + \epsilon_i$.
  - m measures the systematic risk.
    - Also referred to as market risk.
    - It influences all assets in the market to some extent.
  - ϵ_i measures the unsystematic risk.
    - Also referred to as the idiosyncratic risk.
    - It is specific to the company and unrelated to the specific risk of most other companies
    - For different companies $corr(\epsilon_i, \epsilon_j) = 0$.
- Suppose we hold a portfolio composed of N similar assets.
  - $R_P = \frac{1}{N}\left(E(R_1) + \cdots E(R_N)\right) + m + \frac{1}{N}(\epsilon_1 + \cdots + \epsilon_N)$.
  - The last part will be 0 as N gets large enough.
  -