Basics

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Law of circuits

- Kirchhoff's voltage law (KVL): the algebraic sum of all voltages around a loop is zero
  - Steps for KVL
    - Choose name and polarity for each voltage
    - Go around each loop and write KVL
- Kirchhoff's current law (KCL): the algebraic sum of all currents at a node is zero
  - Steps for KCL
    - Choose name and polarity for currents
    - At each node, write KCL
    - Node is between every two elements
- Note:
  - Direction does not matter, stay consistent
  - Voltage drop can be included as a positive or negative sign, so as current
  - Polarity assignments at the beginning are arbitrary
    - Current's polarity does not need to be chosen based on the polarity assigned to the corresponding component
  - The choice of sign convention for writing KVL and KCL in a given circuit are independent

Physical quantities of interest

- Voltage V: potential difference between two points (V)
- Current i: flow rate of charge (A)
- Charge q: a property of matter (C)
- Energy E: (J)
- Power P: time derivative of energy (W)

Node: connection point shared between 2 or more elements
Branch: a segment that contains one element with the two wires sticking out of the two sides
Loop: closed path starting from a given node, going through a number of branches and coming back to the starting point

Common circuit elements:

- Wire: voltage is 0, current can be anything
- Switch:
  - Open: voltage can be anything, current is 0
  - Closed: voltage is 0, current can be anything (wire)
- Resistor: $V = iR$ (Ohm's law).
  - A wire is a resistor with $R \to 0$.
- Capacitor: $i = C\frac{dV}{dt}$.
  - Definition of capacitance C: $q = CV$ (C in Farads).
- Inductors: $V = L\frac{di}{dt}$ (L in Henry).
- Independent voltage source: fixed voltage V_S, any current
- Dependent voltage source: V_S is a function of some other voltage or current in the circuit, current can be anything.
- Independent current source: fixed current I_S, any voltage.
- Dependent current source: I_S is a function of other voltage or current in the circuit, voltage can be anything.
- Diode: current can pass only in one direction
- Volt meter: tells us the voltage between its two sides without disturbing the circuits
  - Connected in parallel with the element whose voltage we want to measure
  - Does not draw any current
- Ammeter: tells us the current through it, without disturbing the circuit
  - connected in series with the element whose current we want to measure
  - Does not take voltage

Electric circuit

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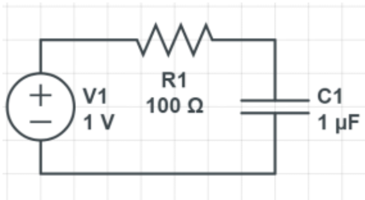
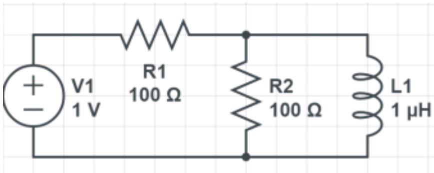
Time constant τ is the time it takes the exponentially-decaying part of the function to reach e^{-1} times the initial value

RC circuit

- 

- $\tau = RC$ is the time constant

RL circuit

- 

- $\tau = \frac{L}{R}$ is the time constant

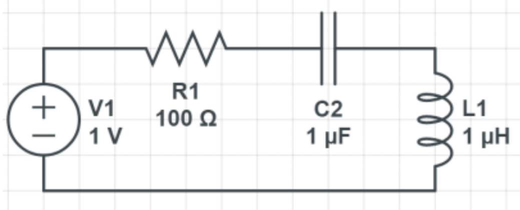
Behavior of capacitors and inductors in DC steady state and during switching in a circuit

- Capacitor
  - DC steady state: things are not charging, time derivative is zero, $\frac{dV}{dt} = 0, i = 0$.
  - During a switching activity: there is no infinite current, the time derivative of the voltage cannot be infinite
    - i.e. V_C cannot change abruptly.
  - Voltage of a capacitor connected to a constant source may not necessarily reach a DC steady state
- Inductor
  - DC steady state: $\frac{di}{dt} = 0, V = 0$ like a short circuit.
  - During a switching activity: i cannot change abruptly.

To solve the transient (time dependent) behavior of a circuit involving capacitors and inductors

- Solve the circuit for its condition before any switching activity
- Determine what happens during the switching and the condition of the circuit immediately after switching
- Use what we found in step 2 as an initial condition for the time evolution of the circuit after switching

2nd order time dependent circuits

- 

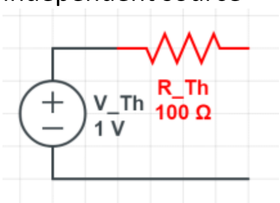
- ODE: $\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0$.
- General solution to ODE ($\frac{d^2 x}{dt^2} + 2\alpha\frac{dx}{dt} + \omega^2 x = \gamma$).
  - $\alpha > \omega$, overdamped $x(t) = m_1 e^{s_1 t} + m_2 e^{s_2 t} + \frac{\gamma}{\omega^2}$, $s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - \omega^2}$.

- $\alpha = \omega$, critical damp $x(t) = (m_1 t + m_2)e^{-\alpha t} + \frac{\gamma}{\omega^2}$.
- $\alpha < \omega$, under damp, $x(t) = e^{-\alpha t}(m_1 \cos(\omega_d t) + m_2 \sin(\omega_d t)) + \frac{\gamma}{\omega^2}$.

Linearity and equivalence

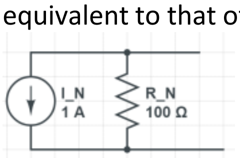
- Linear circuit where the elements have linear behaviors, any voltage or current will end up have a form: $x_j = a_{1j}s_1 + a_{2j}s_2 + \cdots + a_{Nj}s_N$.
- x_j is a voltage/current of interest.
- s_i are all the independent source of voltage and current, and there are a total of N such sources in the circuit.
- By having the answer in such a form, we can now find the new answer very quickly if some of the sources are scaled
- Superposition:
  - x_j has been written as a superposition of the responses of the circuitto the various independent excitations
  - This means that to solve a circuit with multiple independent sources, we can find the response to one independent source at a time by setting all the other independent sources to zero. Repeating this for all independent sources, and adding up all the responses gives the answer
  - Note:
    - Setting a voltage source to zero means replacing it with a wire
    - Setting a current source to zero means replacing it with an open circuit

Thevenin equivalent circuit:

- Theorem: consider a circuit with a number of linear resistors dependant source with linear relationships and independent sources. The behavior of the circuit between a given set of points A and B is equivalent to that of a circuit containing one resistor in series with one independent source
- 

- Short circuit current $I_{sc} = \frac{V_{Th}}{R_{Th}}$.
- Open circuit voltage $V_{oc} = V_{Th}$.
- To find the Thevenin equivalent
  - Find $V_{oc} = V_{Th}$.
  - Then find I_{sc}, and finally R_{Th}.

Norton's equivalent circuit

- Theorem: consider a circuit made of linear resistors, linear dependent sources and independent sources between a given set of points A and B the behavior of the circuit is equivalent to that of a current source in parallel with a resistor
- 

- To find the Norton equivalent
  - Find I_{sc}, then $I_N = I_{sc}$.
  - Find V_{oc}, then $R_N = \frac{V_{oc}}{I_N}$.

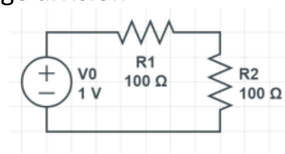
Source transformation (relation between Thevenin and Norton equivalent)

- For a Thevenin and a Norton circuit to be equivalent to each other, they both have to have the same i, v characteristics, which means that they both have the same V_{oc} and I_{oc}.
- Then $R_{Th} = R_N$.

Equivalent resistance

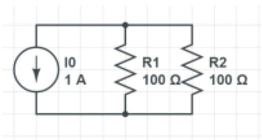
- Series: $R_{eq} = R_1 + R_2 + \cdots + R_N$.
- Parallel: $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$.
- Inductors act similarly to resistors in terms of serial and parallel combinations, capacitors act in the opposite way

Voltage division

- 

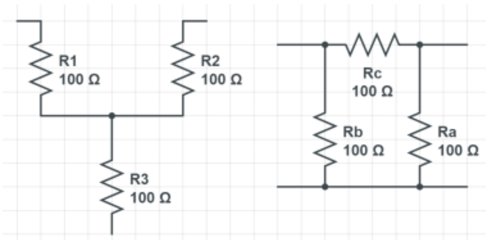
- $V_1 = V_0 \frac{R_1}{R_1 + R_2}$.
- $V_2 = V_0 \frac{R_2}{R_1 + R_2}$.

Current division

- 

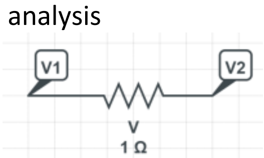
- $i_1 = I_0 \frac{R_2}{R_1 + R_2}$.
- $i_2 = I_0 \frac{R_1}{R_1 + R_2}$.

$Y - \nabla$ transform

- 

- The above two circuits in terms of their behavior as seen from the indicated parts are equivalent if
  - $R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$, $R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$, $R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$.
  - Or $R_a = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_1}$, $R_b = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_2}$, $R_c = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_3}$.

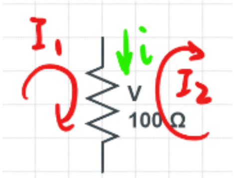
Nodal analysis

- 

- V_1 and V_2 are nodal voltage, $V_1 - V_2 = V$.
- Voltage values are relative to a given reference point such as the ground node.
- KVLs become trivial because we have automatically and implicitly implemented the KVL
- Steps to do nodal analysis
  - Choose a reference node with node voltage 0
  - Define a node voltage for each of the other nodes
  - Write the KCLs for all nodes (except the reference node)
  - Express the currents using node voltages and element relations
- Note: for branches with a voltage source, the current cannot be expressed in terms of node voltages, but each of them gives us an additional equation
- In matrix form $GV = I$:
  - G_{jj} the sum of inverses of resistors directly connected to node j.
  - G_{jk} the negative of the sum of the inverses of resistors directly connected between j and k.

- I_j the algebraic sum of current sources directly connected to node j, with current entering the node being positive.
- V_j are unknowns.

Mesh analysis

- 

- I_1 and I_2 are mesh current, $I_1 - I_2 = i$.
- Steps for mesh analysis:
  - For each mesh (loop with nothing inside), define a mesh current
  - Write all the KVLs (KCLs will be automatically taken care of)
  - Using element relationships, express all voltages in terms of mesh currents
- For current sources, we cannot directly relate their voltage to mesh currents, however, each current source gives us an additional equation
- In matrix forms $I = R^{-1}V$:
  - R_{jj} sum of resistance in mesh j.
  - R_{jk} negative of the sum of the resistances that are part of both meshes j and k.
  - V_j sum of independent voltage sources that are part of mesh j, with a voltage rise in the clockwise direction included with a positive sign.
  - I_j unkown mesh current of mesh j.

Electronic circuit

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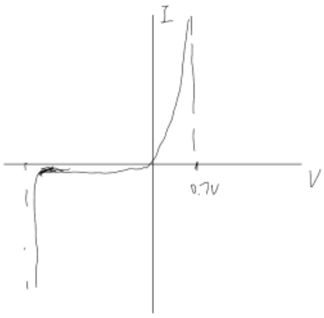
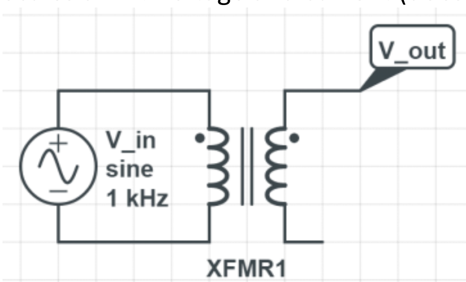
Diode

- Like a switch, on/off determined by voltage
- Current-voltage relation of a real diode
    - 

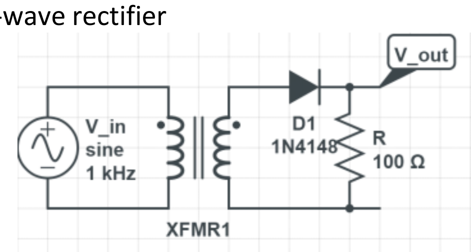
    - In the forward and reverse modes (not including the breakdown), the characteristics can be approximated by $i = I_s\left(e^{\frac{V}{nV_{Th}}} - 1\right)$.
        - I_s is the reverse saturation current.
        - n is a number generally between 1 and 2.
        - V_{Th} is thermal voltave $\frac{k_B T}{e}$, at room temperature $V_{Th} \approx 26mV$.
- Current-voltage relation of an ideal diode
    - No negative current
    - Any current for $V = 0$.
- Simplifications
    - Simplification 1: approximate the forward region by a straight line starting at $0.7V$, going up with some slop, also assume the reverse current is zero
    - Simplification 2: assume that in forward, the straight line at $0.7V$ is vertical, also assume reverse current is zero
    - Simplification 3: assume that in forward, the voltage is 0, also assume the reverse current is zero (ideal diode)

Transformer

- Scales an AC voltage and current (does not add power to the circuit)
- 

- N_l, N_r are the number of turns on the two coils of the transformer.
- $\frac{V_{out}}{V_{in}} = \frac{N_r}{N_l}, \frac{i_{out}}{i_{in}} = \frac{N_l}{N_r}$.

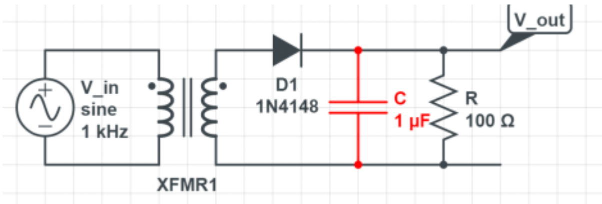
Half-wave rectifier

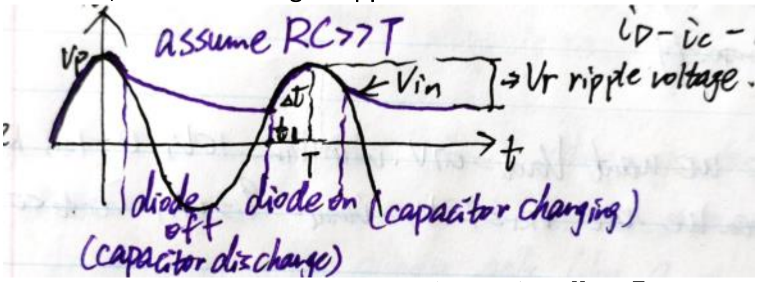
- 

- $V_{out} = \frac{N_r}{N_l} V_{in,peak}\cos(\omega t) - V_D$.

- If diode is on, $V_D = 0$.
- The circuit will only output the amplified positive voltage, all the negative input voltage will output 0 voltage.

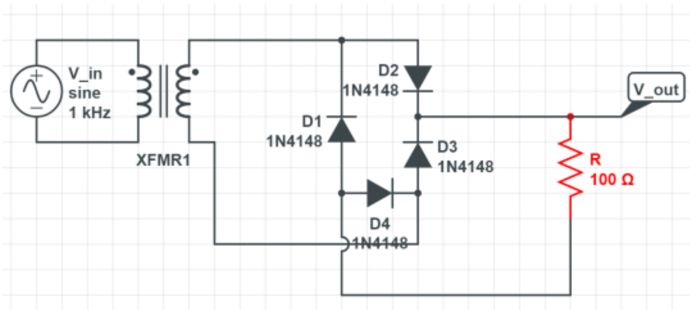
Half-wave rectifier + filter

- 

- Capacitor only discharge to the resistor, because the diode does not allow a current in the opposite direction, so the discharge happens with time constant RC.
- 

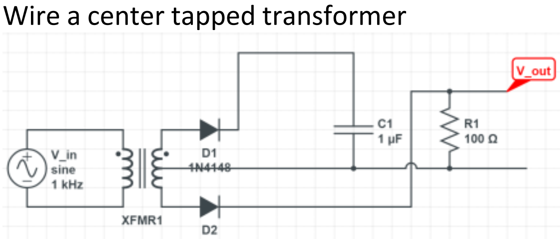
- Ripple voltage: $V_r = V_{peak} - V_{peak}\left(1 - \frac{T}{RC}\right) = \frac{V_{peak}T}{RC}$.
- Average output voltage: $V_{out,avg} = V_{peak} - \frac{V_r}{2}$.
- Diode conduction interval: $\Delta t = \frac{1}{\omega}\sqrt{\frac{2T}{RC}}$.

Full-wave rectifier

- 

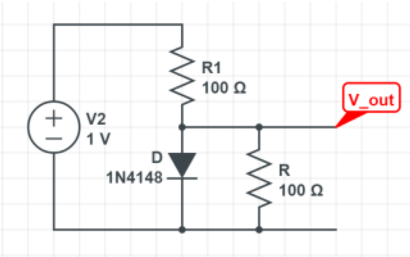
- If V_{in} is positive, 2, 4 are on 1, 3 are off, $V_{out} > 0$.
- If V_{in} is negative, 1, 3 are on 2, 4 are off, $V_{out} > 0$.
- If diodes are realistic, the output will be $V_{in} - 2 \times 0.7V$.

Full wave rectifier with filter

- Wire a center tapped transformer
- 

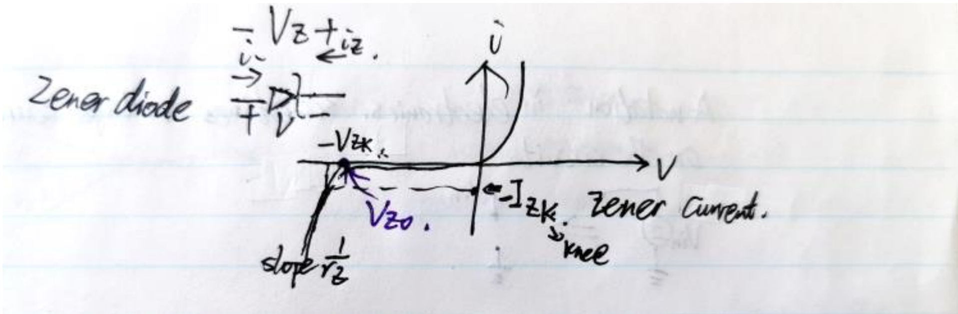
Regulation

- Regulate the output voltage so as to get rid of the ripple as much as possible, we need an element that can help us maintain a more or less fine voltage despite significant variation in the circuit

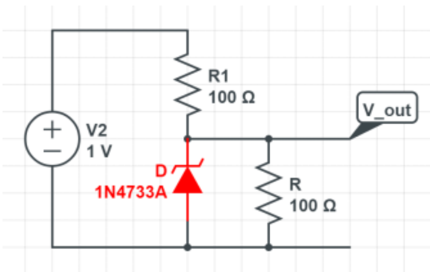
- 

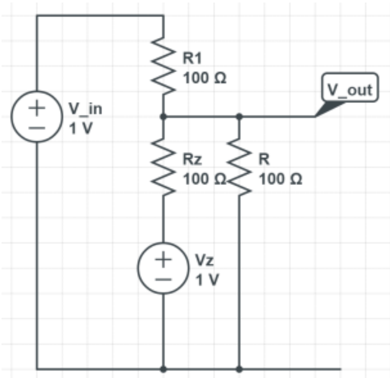
- V_{out} can be $0.7V$ only if the circuit parameters are such that the current through diode i_d can be sufficiently positive.
- Also i_d should not be so high that the diode would be damaged.

Zener diode

- Many diodes get easily damage in reverse break down, but Zener diodes are designed to operate in reverse breakdown
- 

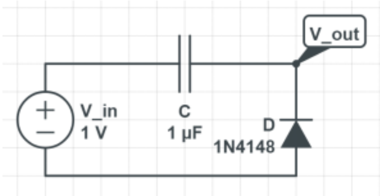
Regulation with Zener diode

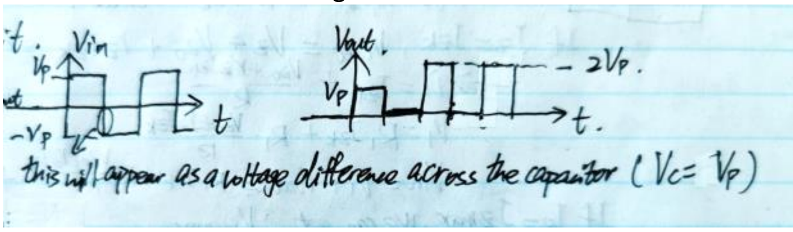
- 

- We need the Zener diode to stay in reverse breakdown as the input fluctuates between two values so that the output voltage is regulated to values near the reverse breakdown voltage of the diode
- Ensure that I_z(current through the Zener diode) stays greater than I_{zk} (the knee current of the Zener diode), but smaller than I_{zmax}(the maximum current the diode can allow in reverse breakdown without being damaged).
- In this case, the diode acts like a voltage source
    - 

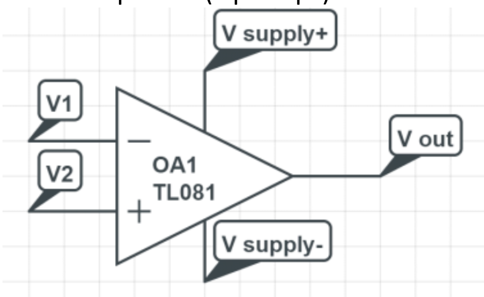
    - For a good Zener diode, R_z is small such that as V_{in} varies between its min and max values, V_{out} changes very little, $V_{out} = V_z + R_z I_z$.

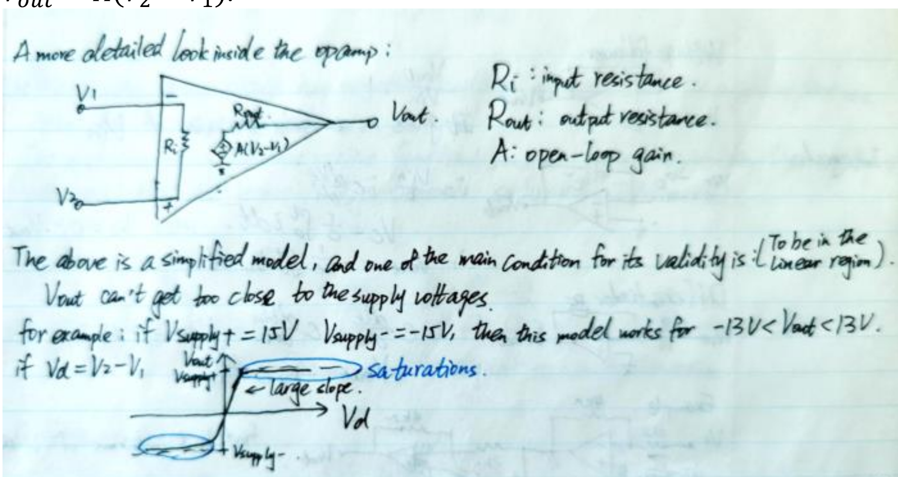
Clamping circuit

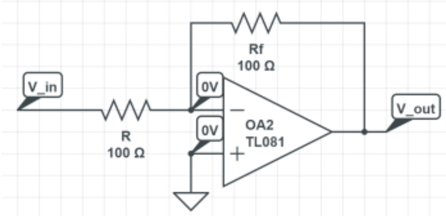
- 

- $V_C = V_{out} - V_{in}$.
- The first time V_{in} goes down below zero, the diode turns on, and the capacitor gets charged to V_p.
- When V_{in} jumps back up, the capacitor cannot discharge, since the diode does not allow a reverse current, so V_{out} will just follow the input with a shift of $V_c = V_P$.
- The diode will not turn on again.
- 

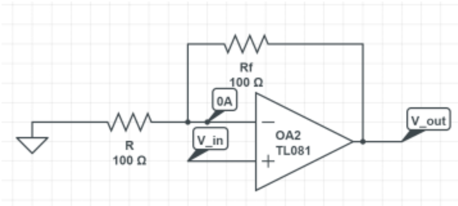
Operational amplifiers (Op Amps)

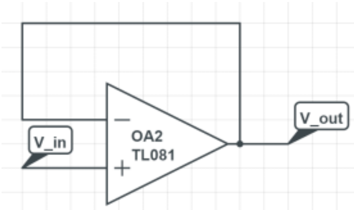
- 

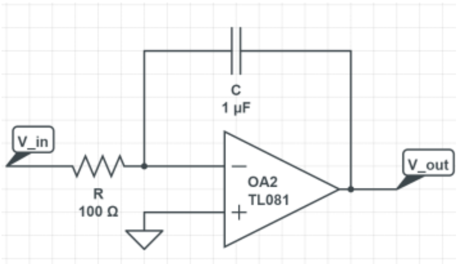
- "-" is the inverting input.
- "+" is the non-inverting input.
- $V_{out} = A(V_2 - V_1)$.
- 

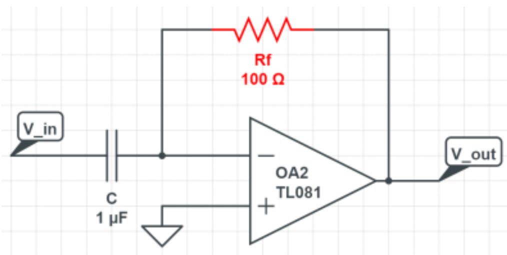
- Ideal op amp
  - $R_{in} = \infty$, $R_{out} = 0$, $A = \infty$.
  - If $V_2 - V_1$ is a small value, the output wants to go to infinity, which means it will saturate
  - Often, we want to stay within the linear region of the characteristics, which means V_{out} should be a finite value in-between the supply voltages
  - Then $V_1 = V_2$.
- Inverting amplifier
  - 

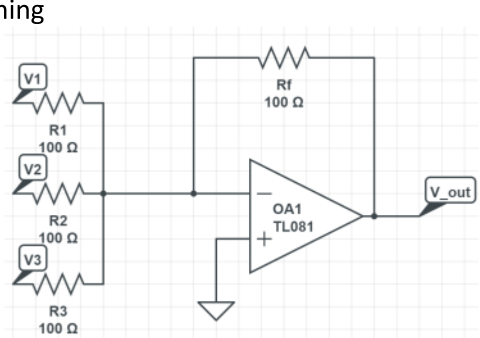
- $\frac{V_{out}}{V_{in}} = -\frac{R_f}{R}$.
- Positive feedback will drive the voltage to saturation

- Non-inverting amplifier
  - 

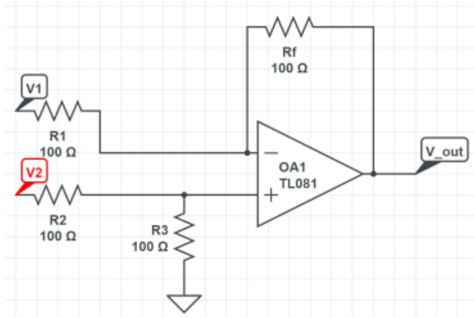
  - $\frac{V_{out}}{V_{in}} = 1 + \frac{R_f}{R}$.
- Voltage follower
  - 

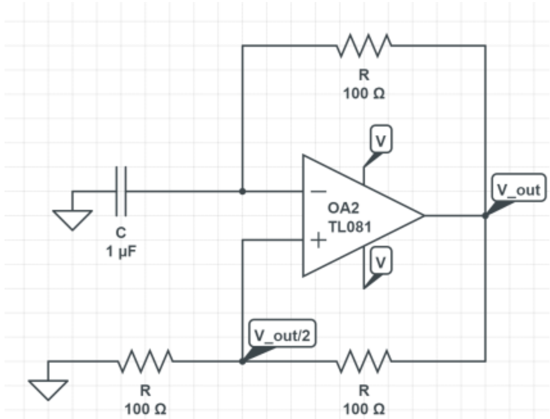
  - $\frac{V_{out}}{V_{in}} = 1$.
- Integrator
  - 

  - $V_{out} = -\frac{1}{RC}\int_0^t V_{in}dt$.
- Differentiator
  - 

  - $V_{out} = -R_f C\frac{dV_{in}}{dt}$.
- Summing
  - 

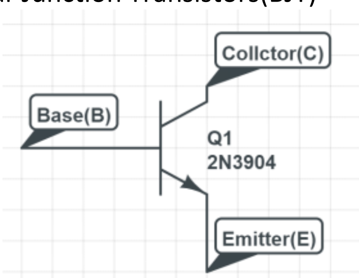
  - $V_{out} = -\left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}\right)R_f$.
- Difference

- 

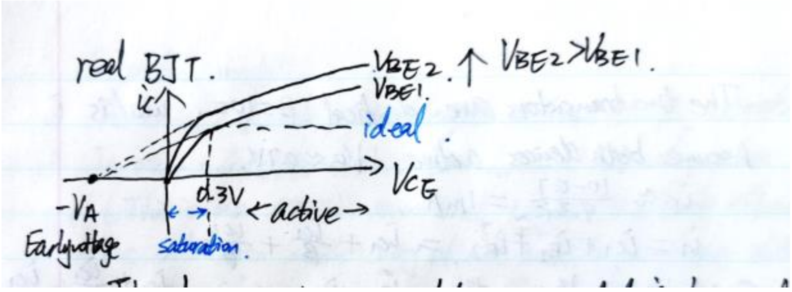
  - $V_{out} = \frac{R_3}{R_2+R_3}\left(1+\frac{R_f}{R_1}\right)V_2 - \frac{R_f}{R_1}V_1$.
- Oscillator
  - 

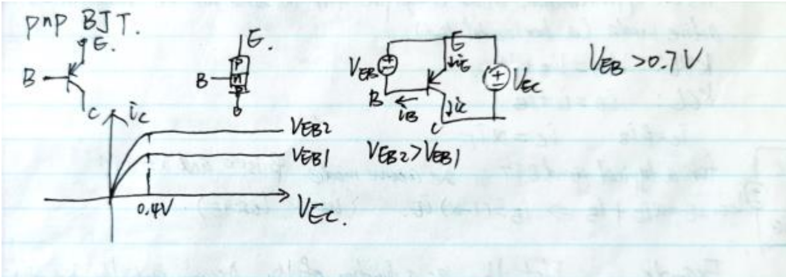
  - Assume at $t = 0$, the capacitor is uncharged
  - A small voltage change at "+" of the op amp, the output will saturate
  - The capacitor starts charging
  - When $V_{amp} = 7.5V$, op amp flips, $V_{out} = -15V$, then it discharges to $-7.5V$, it switches again, V_{out} switches

Bipolar Junction Transistors(BJT)

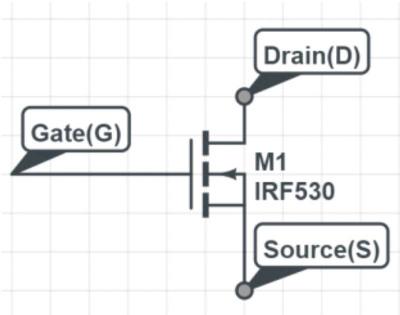
- 

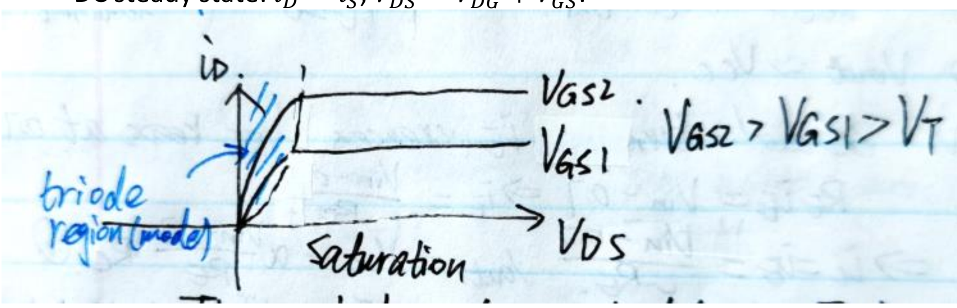
- This is an npn transistor
- $i_E = I_{SE}e^{\frac{V_{BE}}{V_T}}$, $i_C = \alpha i_E$, $\alpha \approx 1$.
  - As long as the collector voltage is not too far below the base voltage $V_C - V_B > -0.4$.
  - Let $\beta = \frac{\alpha}{1-\alpha}$, $i_c = \beta i_B$, β usually of order 100.
  - $i_B = (1-\alpha)i_E$.
- Modes of operation
  - Cut off
    - BE junction is reverse
    - BC junction is reverse
  - Active
    - BE junction is forward
    - BC junction is reverse
  - Saturation
    - BE junction is forward
    - BC junction is forward
- Real BJT

- 

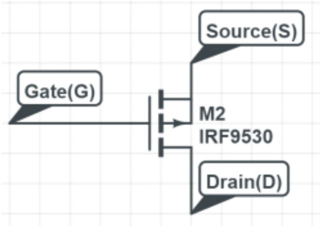
- PNP BJT
  - 

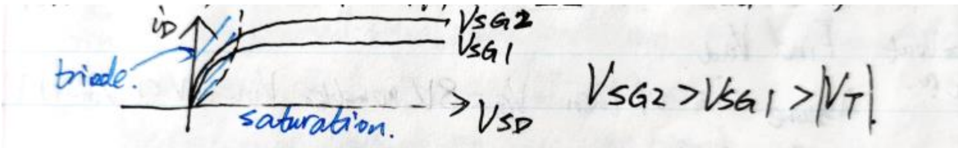
Metal Oxide Semiconductor Field Effect Transistor (MOSFET)

- N-channel MOSFET
  - 

    - DC steady state: $i_D = i_S$, $V_{DS} = V_{DG} + V_{GS}$.
  - 

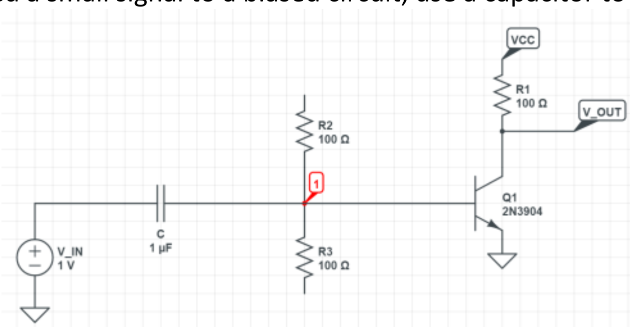
    - The graph ignores the Early effect
  - For the device to turn on, V_{GS} must be greater than a certain threshold voltage ($V_{GS} > V_T$)
  - $i_D = \frac{W}{L}\mu C_{ox}\left((V_{GS} - V_T)V_{DS} - \frac{1}{2}V_{DS}^2\right)$.
    - W is device width, L is channel length.
    - μ is mobility.
    - C_{ox} is oxide capacitance per unit area.
    - This is applicable as long as $0 < V_{DS} < V_{GS} - V_T$.
  - $V_{GS} - V_T = V_{DS}$ corresponds to the dashed line that shows the boundary between the triode and saturation regions in the characteristics graph.
    - At this boundary, $i_D = \frac{W}{2L}\mu C_{ox}\left(V_{GS} - V_T\right)^2$.
  - Saturation mode:
    - For $V_{DS} > V_{GS} - V_T$, the device current stays saturated at $i_D = \frac{W}{2L}\mu C_{ox}\left(V_{GS} - V_T\right)^2$
    - i.e. to first order, i_D does not depend on V_{DS} anymore. (approximation that neglects the Early effect)
    - The saturation mode very often ends up being the desired mode of operation for the MOSFET (like the active mode of the BJT)
- P-channel MOSFET

- 

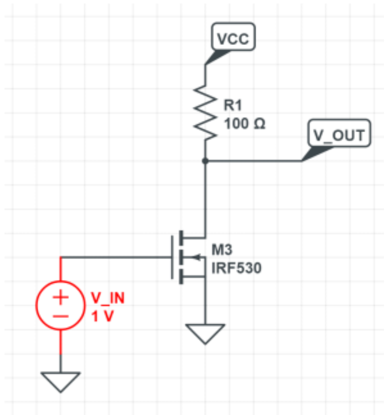
- 

- For the device to be on, $V_{SG} > |V_T|$.
- In triode mode, $0 < V_{SD} < V_{SG} - |V_T|$
  - $i_D = \frac{W}{L}\mu C_{ox}\left((V_{SG} - |V_T|)V_{SD} - \frac{1}{2}V_{SD}^2\right)$.
- In saturation, $V_{SD} > V_{SG} - |V_T|$.
  - $i_D = \frac{W}{2L}\mu C_{ox}(V_{SG} - |V_T|)^2$.

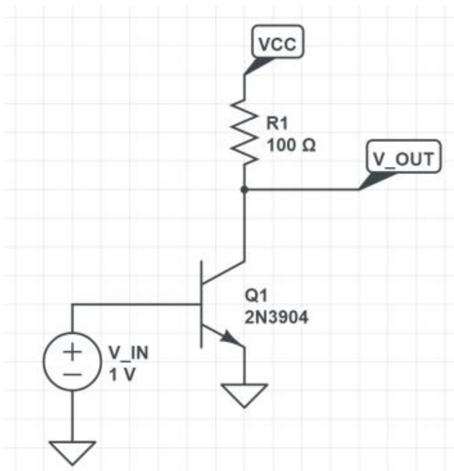
Small signal behavior of transistor circuits

- Let V_{IN} be the complete input voltage consisting of the bias (DC) part of the voltage and the small signal part of the voltage
  - $\frac{V_{OUT}}{V_{IN}} = \frac{V_{OUT}+v_{out}}{V_{IN}+v_{in}}$.
- Goal: come up with a ratio $\frac{v_{out}}{v_{in}}$ that does not depend on v_{in}.
- This is called a small signal voltage gain.
- To feed a small signal to a biased circuit, use a capacitor to couple signals to DC circuits
  - 

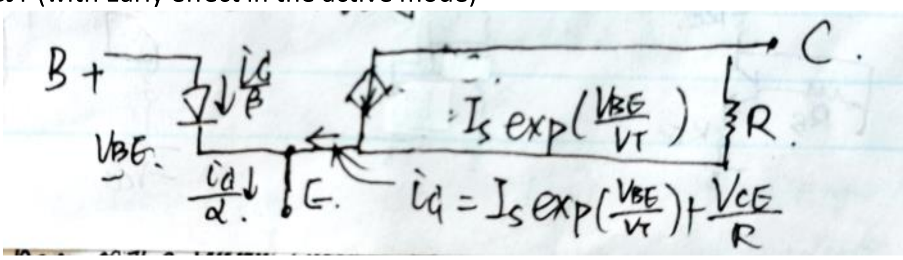
  - At node 1, we see a bias voltage that is determined by the DC behavior of the circuit
  - But if V_{in} is a small signal that varies in time (AC), a good chunk of it will appear at node 1, added to the bias point
- DC and small signal analysis of transistor circuits
  - If we are only concerned with a DC situation, we just have to use the laws of circuits and element relationship
  - If we have small variations of voltages and currents on top of the DC components (bias), we can break the problem down into two parts:
    - A large signal (DC) part
    - A small signal which could be an AC part
    - $v_{IN} = V_{IN} + v_{in}$, where v_{IN} is total signal, V_{IN} is DC component, v_{in} is small signal.
  - Note: the small signal response depends on the bias point, so we first have to solve the bias part
  - Note2: when analyzing small signal behavior, the transistor is not leaving the mode of operation for which we have built out small signal model
- For MOSFET

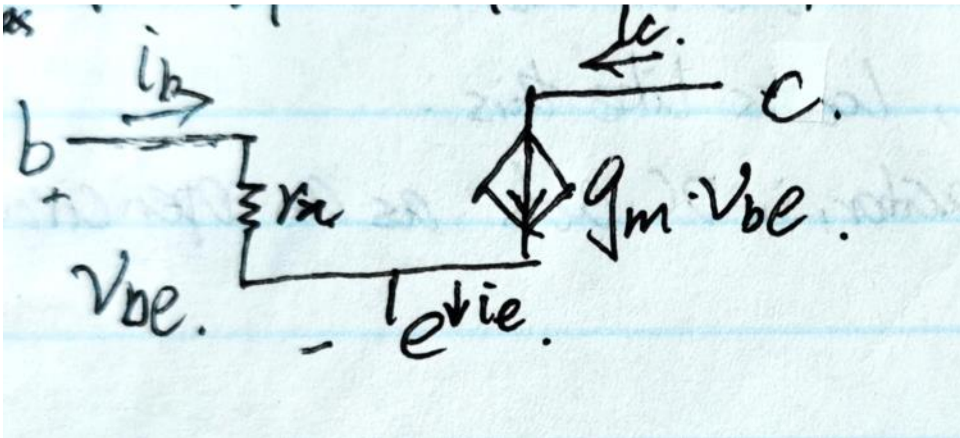
- 

- $i_D = I_D + i_d$.
- $I_D = \frac{1}{2}\frac{W}{L}\mu C_{ox}\left(V_{IN} - V_T\right)^2$.
- $i_d = \frac{W}{L}\mu C_{ox}\left(V_{IN} - V_T\right)v_{in} + \frac{1}{2}\frac{W}{L}\mu C_{ox}v_{in}^2$.
- If v_{in} is small enough, the second term can be neglected.
  - Then $\frac{i_d}{v_{in}} = \frac{W}{L}\mu C_{ox}(V_{IN} - V_T)$.
- $g_m = \frac{i_d}{v_{in}}$ is called small signal transconductance
- Also $\frac{i_d}{v_{in}} = \frac{di_D}{dv_{IN}}$, and $V_{IN} \approx v_{IN}$.
- Small signal input resistance is infinity

• For BJT
  - 

  - Assuming we are biased in the active mode
  - $g_m = \frac{i_c}{v_{in}} = \frac{di_C}{dv_{IN}} = \frac{I_S}{V_T}e^{\frac{v_{IN}}{V_T}}, \frac{v_{out}}{v_{in}} = -g_m R$.

• Small signal models
  - NPN BJT (with Early effect in the active mode)
    - 

- 

    - $g_m = \frac{i_c}{v_{be}}$.
    - $r = \frac{v_{be}}{i_b}$ is the input resistance seen from the base.
    - Both g_m and r depend on the bias point
        - $g_m = \frac{i_c}{v_{be}} = \frac{di_C}{dv_{IN}} = \frac{1}{V_T} I_C$.
        - $r = \frac{dv_{BE}}{di_B} = \frac{v_{be}}{i_b} = \frac{\beta}{g_m}$.