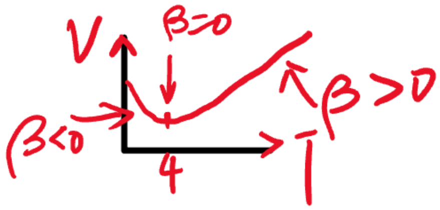
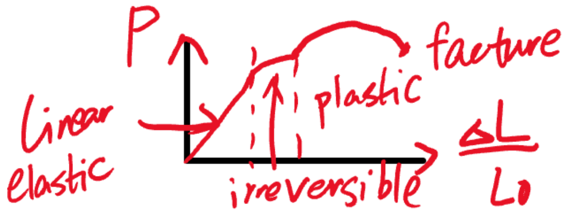
Thermodynamics

2019年6月22日 9:29

- Conservation of energy
  - Each part of a physics system has a certain amount of energy
  - The total energy of an isolated system does not change with time
  - Energy can move between different parts and take different forms
- In thermodynamics, we care about microscopic kinetic and potential energy of atoms/molecules
  - When we heat/cool an object, we are adding/removing energy at the molecular level
  - The size, volume and other macroscopic characters may slightly change
- Kelvin scale
  - Defined by T=constant × pressure
    - i.e. P is proportional to T
  - T=273.16K at triple point of water
- Thermal expansion
  - Materials usually expand when heated due to higher atom energy, thus further distance between atoms
  - Linear expansion: $\Delta L = \alpha L_0 \Delta T$
  - Volume expansion: $\Delta V = \beta V_0 \Delta T$, where β=3α for solids
    - Water is a special example



  - Thermal expansion together with expansion/compression due to mechanical forces
    - $\frac{F}{A} = Y\frac{\Delta L}{L_0}$, Y is the Young's modulus, only valid when $\frac{\Delta L}{L_0}$ is small.

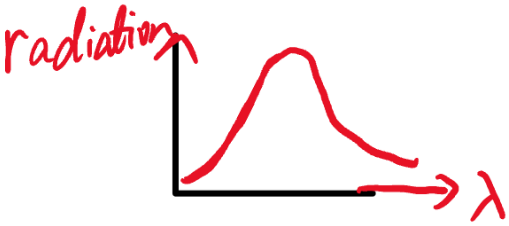


    - Young's modulus is a basic property of a material. It only depends on material.
      - The larger the Y value, the harder the material
  - Thermal stress: forces on a material preventing expansion/contraction due to heating/cooling
    - $\frac{F}{A} = -Y\alpha\Delta T$ (assuming zero change in length)
- Heat(Q): amount of energy transferred due to temperature differences.
  - Specific heat (c_x in $\frac{J}{kg \cdot K}$): energy required to heat 1 kg of material by 1 K
    - $Q = mC\Delta T$
  - Molar specific heat(C_x in $\frac{J}{mol \cdot K}$): energy required to heat 1 mol of material by 1 K
    - $Q = nC\Delta T$
- Phase of matter
  - Critical point: little density changes
  - Macroscopic properties change dramatically across phase boundary
  - At transition temperature, transition occurs due to heat added/removed with no temperature change
  - Latent heat (L in $\frac{J}{kg}$): amount of heat required for transition per mass of material
    - $Q = mL$
    - L_f if the latent heat for freezing/melting
    - L_v if the latent heat for boiling/condensing
- Thermal conductivity
  - Heat moves more quickly through some materials than others in response to a temperature gradient
  - Poor thermal conductor is called insulator (e.g. plastic)
  - Thermal conductivity determines heat current from temperature gradient

- $H = kA\frac{dT}{dx} = kA\frac{T_H - T_c}{L}$
  - k is the thermal conductivity. It is a basic property of a material
  - H is the heat current in $\frac{J}{s}$

- Thermal resistance
  - Measures the effectiveness of insulation layer
  - $R = \frac{L}{k}$

- Heat transfer
  - Convection: heat transfer via macroscopic motion of fluids
  - Radiation: all objects give off electromagnetic radiation which carries energy away from the object
    - Thermal radiation from an object is typically in IR/visible
    - We can measure energy current at various wavelengths-spectrum
    - Peak wavelength is inversely proportional to T
      - $\lambda_{max} = \frac{b}{T}$, b=2.9K•mm (Wien displacement law)
      - Outer space peak at 1mm, T=2.7K



    - Total power from thermal radiation
      - $H = Ae\sigma T^4$
        - $\sigma = 5.67 \times 10^{-8} \frac{W}{m^2 \cdot K^4}$ (Stefan-Boltzmann constant)
        - $e = 1$ for perfect absorber(black body)
          - Emits the most thermal radiation for a given T
        - $e = 0$ for perfect reflector
        - $e = \frac{H}{H_{black\ body}}$ for other objects
      - Equilibrium⇔no heat current⇔ $H_{in} = H_{out}$
    - Power per unit area (Intensity) at distance *R* from an object
      - $I = \frac{H}{4\pi R}$
      - Solar constant $I_{SC} = 1367 \frac{W}{m^2}$
      - $H_{in} = \pi r^2(1-a)I$ for an object at distance *R* from a radiation source with radius *r*
        - a is the reflectal fraction

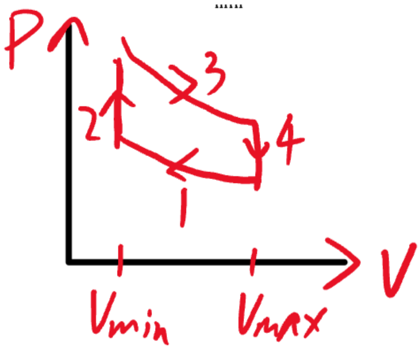
- First law of thermodynamics
  - $\Delta U = Q - W$
    - Q is the heat added to gas
    - W is the work done by gas
      - $W = F\Delta x = PA\Delta x = P\Delta V = \int_{v_i}^{v_f} P(V)\,dV$
      - Compression: W is negative

- Ideal gas law
  - Microscopic: $P = const \cdot \frac{N}{V} m v_{avg}^2$
    - N is the number of molecules
  - Macroscopic:$PV = nRT$
    - R=8.31$\frac{J}{mol \cdot K}$

- Energy and energy change of a gas
  - $\Delta U = nC_v\Delta T$
    - It only depends on initial and final state
    - C_v is the molar specific heat
      - $C_v = \frac{3}{2}R$ for ideal monatomic gases, $\frac{5}{2}R$ for ideal diatomic gases
  - Constant volume (isochoric)
    - $\frac{T_2}{T_1} = \frac{P_2}{P_1}$

        - $W = \Delta V = 0$
        - $Q = \Delta U = nC_v\Delta T$
    - Constant pressure (isobaric)
        - $Q = \Delta U + W = \Delta U + P\Delta V = \Delta U + nR\Delta T = n(R + C_v)\Delta T$
            - $C_p = R + C_v$
    - Constant temperature (isothermal)
        - $P_1V_1 = P_2V_2$
        - $\Delta U = 0$
        - $Q = W = \int_{v_1}^{v_2} P(V)\,\mathrm{d}V = nRTln\left(\frac{V_2}{V_1}\right)$
    - Adiabatic
        - $Q = 0$
        - $\Delta U = -W$
        - $PV^\gamma = const, \gamma = \frac{C_p}{C_v}$
        - $TV^{\gamma-1} = const$

- Heat engines
    - Partially convert heat to work via cyclic process
    - Efficiency: $e = \frac{W}{Q_{ab}} = 1 - \frac{Q_{ex}}{Q_{ab}}$
    - Internal combustion engine
        - Step 1: adiabatic compression
        - Step 2: combustion of gasoline (heating at constant volume)
        - Step 3: adiabatic expansion
        - Step 4: exhaust out
        - $e = \frac{r^{\gamma-1} - 1}{r^{\gamma-1}} = 1 - \frac{1}{r^{\gamma-1}}$
            - r is the compression ratio, higher efficiency for larger r, but gasoline will spontaneously ignite if r is too large. $r = \frac{V_{\max}}{V_{\min}} \in (8,10)$

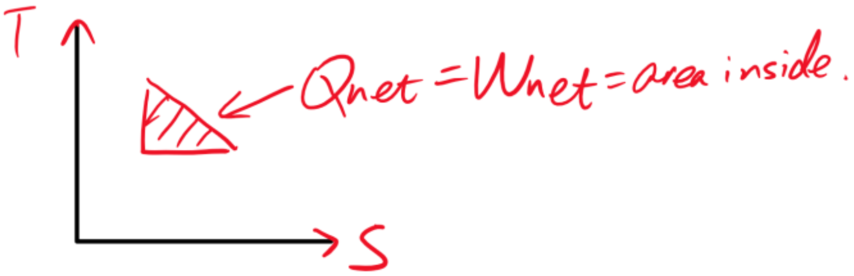


    - Refrigerators (heat engine in reverse)
        - Can transfer heat from colder system to warmer system by doing work
        - Step 1: constant temperature expansion
        - Step 2: constant volume heating
        - Step 3: constant temperature compression
        - Step 4: constant volume cooling

- Entropy and second law of thermodynamics
    - Energy is exchanged between nearby molecules via random process
    - Entropy is a measure of many possible microscopic configurations there are for a specified set of macroscopic configurations

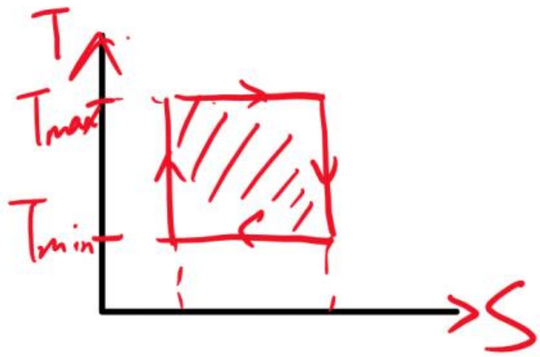
- Define entropy of a macroscopic configuration to be $S = const \times \ln N$
  - $dS = \frac{dQ}{T}$
    - $dS = 0$ for adiabatic process
    - $dS = \frac{dQ}{T}$ for isothermal
    - $\Delta S(total\ change) = 0$ for a closed cycle
  - Entropy is a state variable and has additivity $(S = S_1 + S_2 + \cdots)$

S increases⇒ $Q > 0$
S decreases⇒ $Q < 0$



- Second law of thermodynamics
  - Total entropy of a system always increases, (probability of decrease is unimaginably small)

- Carnot cycle
  - $e = 1 - \frac{T_{\max}}{T_{\min}}$
    - It is the maximum possible efficiency at a fixed $T_{\max}$ and $T_{\min}$

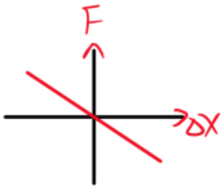


⇒

Oscillations & Waves

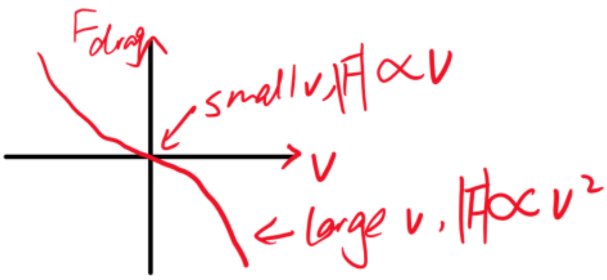
2019年6月25日 17:12

- **Mechanical equilibrium** occurs when forces (and torques) on each part of the system add to zero
- **Restoring force**
  - For a stable equilibrium configuration, a displacement in on direction leads to a net force in the other direction
  - Hooke's law: ideally $F = -k\Delta x$. This applies to almost any system pertubed a small amount from stable equilibrium



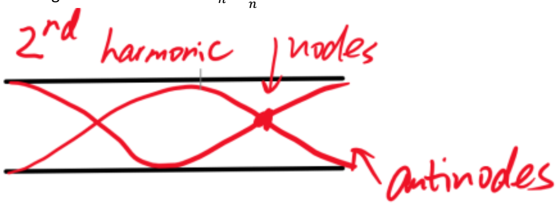
- **Simple harmonic motion**
  - $F = -k\Delta x = ma \Rightarrow \frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = a = -\frac{k}{m}x$
    - Δx is always the total displacement from equilibrium point
  - General function:
    - $x(t) = A\cos(\omega t + \varphi)$
    - $v(t) = -A\omega\sin(\omega t + \varphi)$
    - $\omega = \sqrt{\frac{k}{m}} = \frac{2\pi}{T} = 2\pi f$
  - Frequency: $f = \frac{1}{T}$
    - In Hz or s^{-1}
  - Work and energy
    - Work done on the mass by spring $W = \frac{1}{2}k\Delta x^2$

    - Potential energy of spring $PE = \frac{1}{2}k\Delta x^2$
    - Kinetic energy $KE = \frac{1}{2}mv^2$
    - Total energy for a horizontal system $U = PE + KE = \frac{1}{2}k\Delta x^2 + \frac{1}{2}mv^2$ is conserved
    - Total energy for a vertical system $U = PE + KE = mgh + \frac{1}{2}k\Delta x^2 + \frac{1}{2}mv^2$ is conserved
- **Real oscillators**
  - Energy is lost becoming thermal energy
    - Amplitude decreases with time by the same fraction $A = A_0 r^{-\frac{t}{T}}$
  - Forces that lead to damping
    - Velocity dependent & opposite direction to velocity



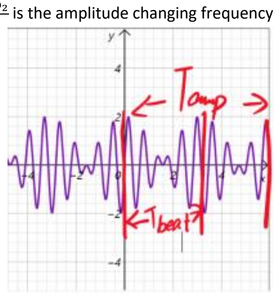
    - Vicious fluid drag: $F_D = -bv$
  - Velocity change is related to k, m, x, b, v
    - $\frac{dv}{dt} = -\frac{k}{m}x - \frac{b}{m}v$
  - Damped oscillations:
    - $x(t) = A_0 e^{-\frac{t}{T}}\cos(\omega t + \varphi)$
      - $T = \frac{2m}{b}$
      - $\omega = \sqrt{\frac{k}{m} - \frac{b^2}{4m}}$ valid for $b < \sqrt{2km}$
        - When $b = \sqrt{2km}$, critical damping (pure exponential decay with no oscillation)
        - When $b > \sqrt{2km}$, exponential decay slower to reach equilibrium than critical damping
        - To make a good shock absorber, $b < \sqrt{2km}$(responds faster)
  - Quality factor:
    - $Q = \frac{2\pi}{1 - \left(\frac{A_1}{A_0}\right)^2}$
    - Larger Q is smaller damping

- Forced oscillations and resonance
  - We can add in an oscillating force by hand
    - $F = F_0 \cos(\omega_0 t)$
    - ω_0 is the driving frequency controlled by hand
  - Resonance
    - When $\omega_0 = \omega = \sqrt{\frac{k}{m}}$, amplitude will be the largest
    - $A_{\max} = \frac{F_0}{\omega b}$
- Waves
  - Travelling waves
    - Transverse wave (slower): oscillations perpendicular to travelling directions
      - Wave speed on a string: $v = \sqrt{\frac{F}{\mu}}$
        - F is the tension force
        - μ is mass per unit length (linear mass density) of the string
    - Longitudinal wave (faster): oscillations parallel to travelling directions
  - Picture of waves
    - At an instance: $f(x) = A\cos(kx + \varphi)$, $k = \frac{2\pi}{\lambda}$
    - General formula:
      - Moving to the right: $f(x,t) = A\cos(kx - \omega t)$
      - Moving to the left: $f(x,t) = A\cos(kx + \omega t)$
      - $\omega = \frac{2\pi}{T}, k = \frac{2\pi}{\lambda}, v = \frac{\omega}{k}$
  - Power and energy of waves
    - Energy $\propto A^2$(amplitude)
    - Energy intensity decreases with distance for spherical or surface waves ($\propto \frac{1}{r^2}$)
    - Power of a wave $P = \frac{1}{2}\mu A^2\omega^2 v$
  - Standing waves (systems with boundaries)
    - Nodes: at which the string never moves
    - Antinodes: at which the amplitude of string motion is the greatest
    - Longitudinal standing waves (Kundt's tube)
      - Displacement node = pressure antinodes = nodes
    - Standing waves in a tube with two open ends
      - Pipe's end is always an antinode
      - Frequency for nth harmonic (n nodes): $f_n = n\frac{v}{2L}$
      - Wavelength for nth harmonic: $\lambda_n = \frac{2L}{n}$



    - Standing waves in a tube with one open end
      - Pipe's closed end is always a node
      - Frequency: $f_n = n_{odd}\frac{v}{4L}$ $(n_{odd} = 1,3,5,7,\ldots)$
      - Wavelength: $\lambda_n = \frac{4L}{n_{odd}}$

    - nth overtone is $n + 1$th harmonic
  - Beats
    - Slightly mismatched frequencies cause audible beats
      - In phase⇔interfere constructively⇔loud
      - (half cycle)out of phase⇔interfere destructively⇔soft
    - General formula if $y_1 = A\sin(\omega_1 t)$, $y_2 = A\sin(\omega_2 t)$
      - $y = y_1 + y_2 = 2A\cos\left(\frac{\omega_1 - \omega_2}{2}t\right)\sin\left(\frac{\omega_1 + \omega_2}{2}t\right)$
        - $\frac{\omega_1+\omega_2}{2}$ is the oscillation frequency (what we hear)
        - $\frac{\omega_1-\omega_2}{2}$ is the amplitude changing frequency

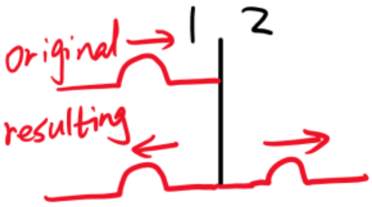


        - $T_{beat} = \frac{1}{2}T_{amp}$ is the time for the sound to go from loud to soft to loud
        - $f_{beat} = f_1 - f_2$
  - Superposition of waves
    - $y_{total}(x,t) = y_1(x,t) + y_2(x,t) + \cdots$
    - If $y_1(x,t) = A\cos(kx - \omega t)$ and $y_2(x,t) = A\cos(kx + \omega t)$ then the resulting wave
      - $y = 2A\sin(kx)\cos(\omega t)$
      - Is a standing wave, $v = 0$

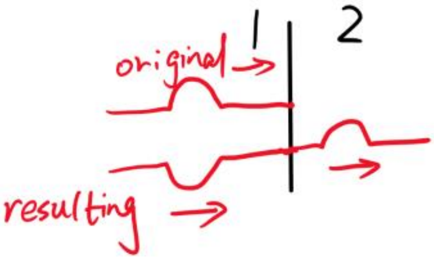
- Interference
    - Must have the same frequency
    - Constructive
        - Path difference: $r_1 - r_2 = m\lambda\ (m = 0,1,2 \ldots)$
        - Phase difference: $k\Delta x + \Delta\varphi = 2\pi n\ (n = \cdots, -1,0,1,2, \ldots)$
    - Destructive
        - Path difference: $r_1 - r_2 = (m + \frac{1}{2})\lambda\ (m = 0,1,2 \ldots)$
        - Phase difference: $k\Delta x + \Delta\varphi = n\pi\ (n = \cdots, -3, -1,1,3, \ldots)$

- Light
    - $c = 3 \times 10^8 \frac{m}{s}$
    - Refraction
        - Speed of lights are different in different media: $v = \frac{c}{n}$
        - n is the refraction rate, in any medium other than vacuum $n > 1$
        - Snell's law: $n_1 \sin\theta_1 = n_2 \sin\theta_2$
            - 

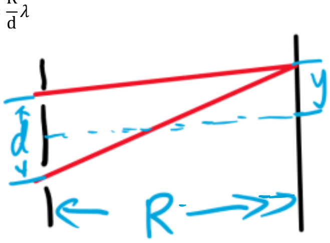
    - Reflection
        - It is due to the difference between refraction rates of two media
        - When $n_1 = n_2$, no reflection
        - When $n_1 > n_2$, wave will not be inverted



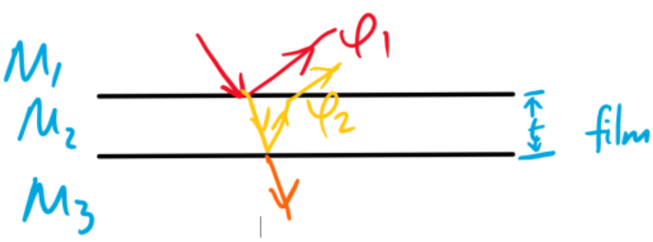
        - When $n_1 < n_2$, wave will be inverted, adding a phase π



    - Double split interference
        - $r_1 - r_2 = d\sin\theta$
        - $\Delta y = \frac{\mathrm{R}}{\mathrm{d}}\lambda$



    - Thin film interference



        - Constructive interference: $k\Delta x + \Delta\varphi = 2\pi n\ (n = \cdots, -1,0,1,2, \ldots)$
        - Destructive interference: $k\Delta x + \Delta\varphi = n\pi\ (n = \cdots, -3, -1,1,3, \ldots)$
            - Here $\Delta x = 2\mathrm{t}, k = \frac{2\pi}{\lambda_{film}}, \lambda_{film} = \frac{\lambda_{air}}{n_{film}}, \Delta\varphi$ is due to reflection
        - No transmission⇔maximum reflection⇔constructive interference
        - No reflection⇔maximum transmission⇔destructive interference

Electric circuits & components

2019年6月26日 17:34

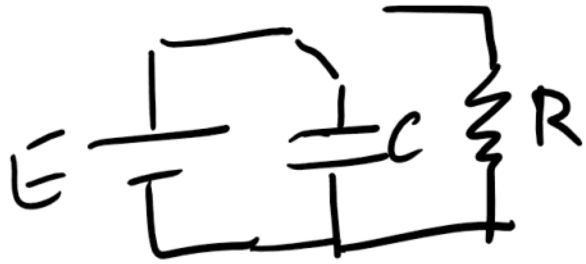
- Resistors: $V = IR$
- ## Capacitors
  - 

  - $Q = C\Delta V$
    - C in F(arad)
  - Compare with batteries
    -

Capacitors	Batteries
Not a source of potential (V changes quickly)	Constant potential difference
High voltage	Low voltage
No resistance	Internal resistance

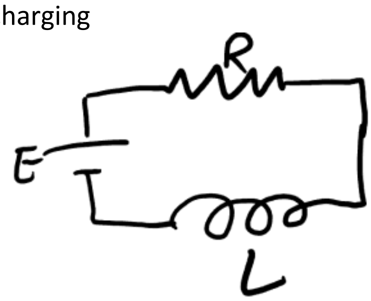
  - Properties
    - In series
      - Each capacitor has the same Q
      - $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$
    - In parallel
      - Each capacitor has the same ΔV
      - $C_{eq} = C_1 + C_2 + \cdots$
  - Energy stored in capacitors
    - $U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}C\Delta V^2$
- ## Inductors
  - 

  - $V_L = -L\frac{di}{dt}$
    - L is the inductance in H(enry)
    - $L = \frac{N\phi_B}{I}$, N is the number of turns
  - Properties
    - In series
      - $L_{eq} = L_1 + L_2 + \cdots$
    - In parallel
      - $\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots$
    - Stores energy in a magnetic field
    - Resists changes (currents cannot change immediately)
  - Energy stored in inductors
    - $U = \frac{1}{2}LI^2$
- ## Direct current (DC)circuits
  - R-C circuits

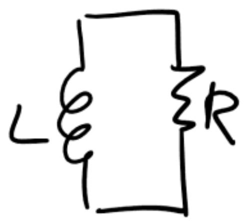


- Time constant $\tau = RC$
- Charging
  - $I(t) = I_0 \mathrm{e}^{-\frac{t}{RC}}$
  - $Q(t) = CV\left(1 - \mathrm{e}^{-\frac{t}{\mathrm{RC}}}\right)$
- Discharging
  - $I(t) = I_0 \mathrm{e}^{-\frac{t}{RC}}$
  - $Q(t) = CV\mathrm{e}^{-\frac{t}{RC}}$

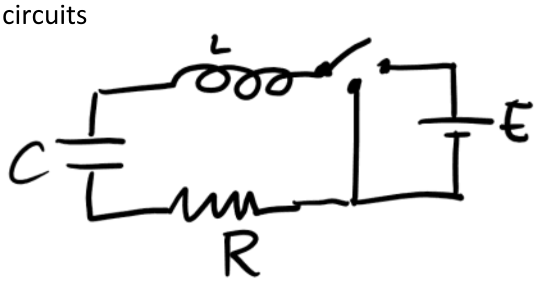
- R-L circuits
  - Time constant $\tau = \frac{L}{R}$
  - charging



    - $I(t) = I_{\max}\left(1 - \mathrm{e}^{-\frac{Rt}{\mathrm{L}}}\right)$
  - Discharging



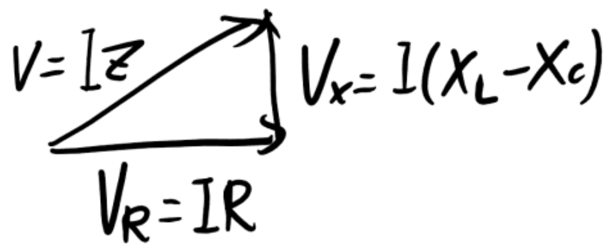
    - $I(t) = I_0 \mathrm{e}^{-\frac{Rt}{\mathrm{L}}}$
- R-L-C circuits



  - $q(t) = q_0 \mathrm{e}^{-\lambda t} \cos(\omega t + \varphi)$
    - $\lambda = \frac{\mathrm{R}}{2L}$
    - $\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$
      - When $\omega > 0$ oscillatory underdamped
      - When $\omega = 0$ critically damped
      - When $\omega = a + bi$ overdampe(exponential decay)

- Let $R \to 0$, the circuit become an L-C circuit
  - $\omega = \sqrt{\frac{1}{LC}}$
  - $q(t) = q_0 \cos(\omega t + \varphi)$
  - $I(t) = |\frac{dq}{dt}| = \omega q_0 \sin(\omega t + \varphi)$

- Alternating Current (AC) circuit
  - $V(t) = V_{max} \sin(\omega t + \varphi)$
  - $I(t) = I_{max} \sin(\omega t + \varphi)$
  - In North America $f = 60Hz$
  - Root mean square values
    - This is what meters measure
    - $V_{Rms} = \frac{V}{\sqrt{2}}$
    - $I_{Rms} = \frac{I}{\sqrt{2}}$
  - Purely "R" resistive
    - Reactance $X_R = R$
    - V in phase with I
    - $I(t) = \frac{V}{R} \sin(\omega t + \varphi)$
  - Purely "L" inductive
    - Reactance $X_L = \omega L$
    - I achieves its max after V,V leads I
    - $I(t) = \frac{V}{\omega L} \sin\left(\omega t + \varphi - \frac{\pi}{2}\right)$
  - Purely "C" capacitive
    - Reactance $X_C = \frac{1}{\omega C}$
    - I achieves its max before V,V lags I
    - $I(t) = V\, \omega C \sin\left(\omega t + \varphi + \frac{\pi}{2}\right)$
  - R-L-C circuits
    - Impedance triangle



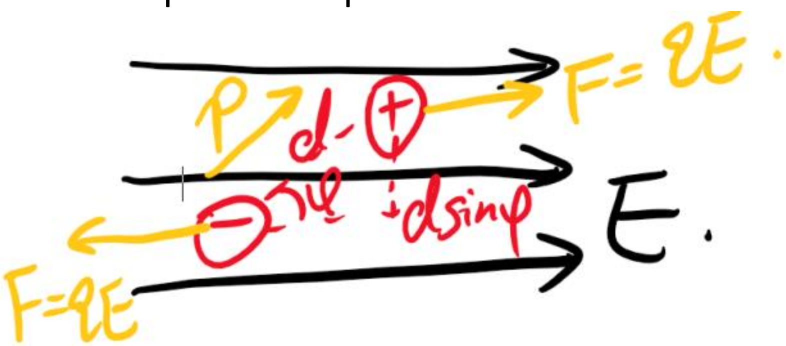
      - Total impedance $Z = \sqrt{{X_R}^2 + (X_L - X_c)^2}$
      - It will rotate around the origin
      - $I_{max} = \frac{V}{Z}$
  - Power dissipation
    - Power can only be dissipated in R
    - Power dissipation in inductor/capacitor has both positive and negative power in one cycle which cancel out
    - When $\varphi = 0$,the average power dissipated in R is $P = \frac{1}{2} I_{max} V_{max} = I_{RMS} V_{RMS}$
    - Otherwise,$P = \frac{1}{2} I_{max} V_{max} \cos(\varphi) = I_{RMS} V_{RMS} \cos(\varphi)$
      - $\cos(\varphi)$ is the power factor
  - Resonance in AC circuits

    - $I = \frac{V}{Z} = \frac{V}{\sqrt{{X_R}^2 + (X_L - X_c)^2}}$ is a function of ω
    - When $X_L = \omega L = X_C = \frac{1}{\omega C}$, R is the minimum, and $I = \frac{V}{R}$ is the maximum
  - Treat parallel AC circuits as combined series AC circuits

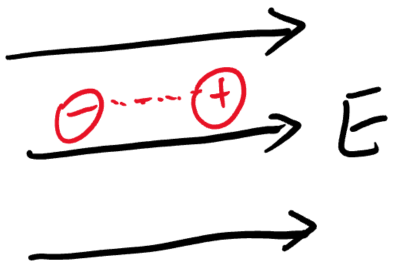
Electricity & magnetism

2019年6月26日 17:35

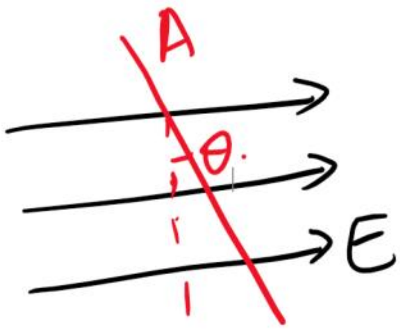
- Coulomb's Law
    - $\left|\vec{F}\right| = k\frac{|q_1||q_2|}{r^3}|\vec{r}| = k\frac{|c||q_2|}{r^2}$
    - $k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\, {Nm^2}/{C^2}$
- Electric fields and superposition
    - $E = \frac{F}{q}$
    - $E_{net} = \sum \frac{k \cdot Q_i}{r_i^2}\hat{r}_i$ is a vector sum
    - E-field always points in the direction that a positive test charge would move
    - Field vectors are tangent to the field lines
    - Field line start on positive charges and ends on negative charges
    - Density of field lines show the field strength
    - Use integrals for charges on a ring or a bar
- Force and torque on a dipole



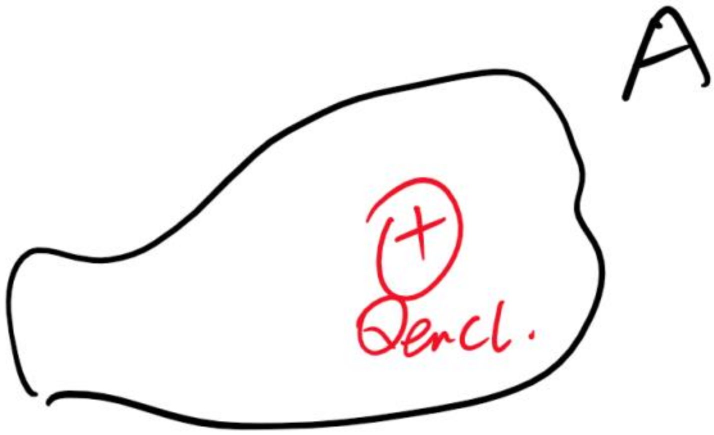
    - Stable position



    - p is the dipole moment
    - $\vec{p} = q \cdot \vec{d}$ direction from - to +
    - Torque: $\vec{\tau} = \vec{F} \times \vec{r} = qE\,\mathrm{d}\sin\varphi = pE\sin\varphi = \vec{p} \times \vec{E}$
    - Potential energy $U_E = -\vec{p} \cdot \vec{E} = -PE\cos\varphi$
- Gauss's Law
    - Benefits
        - Easier for symmetry circumstances
        - Better understanding of conductors in electrostatic equilibrium
        - Valid for moving charges
    - Electric flux passing through a surface $\phi_E = EA\cos\theta$



- Electric flux passing through a closed Gaussian surface
  - $\phi_E = \oiint \vec{E} \cdot \mathrm{d}\vec{A} = \oint \vec{E} \cdot \vec{n} \, \mathrm{d}A = \frac{Q_{enclosed}}{\varepsilon_0}$

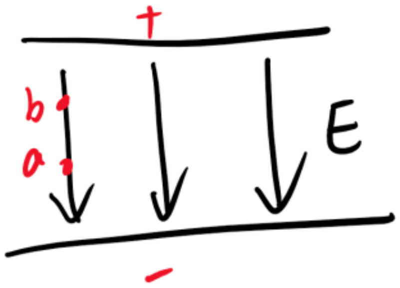


- Gauss's Law:
  - The net flux passing through a closed surface is proportional to the net charge inside the surface.
  - It does not depend on the shape of the surface.
  - Define flux passing out of the closed Gaussian surface to be positive
- $\phi_{total} = \frac{1}{\varepsilon_0}(Q_1 + Q_2 + \cdots)$
- Applications of Gauss's Law
  - Point charges: $E = \frac{q}{4\pi r^2 \varepsilon_0}$
  - inside a spherical conducting shell: $E = 0$
  - Inside an insulator use integral to find $Q_{enclosed}$
  - Uniform line charge with charge density λ: $E = \frac{\lambda}{2\pi r \varepsilon_0}$
  - Uniformly charged plane sheet with charge density σ: $E = \frac{\sigma}{2\varepsilon_0}$
    - Between two opposite charged parallel plates: $E = \frac{\sigma}{\varepsilon_0}$

Conductors

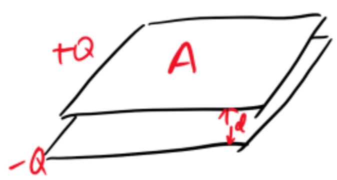
- Materials in which charges are free to move
- Zero net charge, thus $E = 0$ inside a conductor.
- All excess charges are distributed over the surface (may be non-uniform)
  - E.g. the inner surface has -5μC non-uniform charge distribution, the outer surface has 5μC uniform charge distribution

Work and potential in electric fields



- $W = qEd = \int \vec{F} \cdot \mathrm{d}\vec{r}$ $(in\ 3D)$
- For process a→b
  - $W_{ext} = U_b - U_a$
  - $W_{E-field} = -(U_b - U_a)$
- Work is independent of path, all forces are conservative
- U is the potential energy
- Electric potential difference
  - $\Delta V = \frac{\Delta U}{q} = \Sigma\, \mathrm{d}V = -\Sigma \vec{E} \cdot \mathrm{d}\vec{l} = -\int_a^b \vec{E} \cdot \mathrm{d}\vec{l}$
  - Potential for a point charge $V = \frac{kq}{r}$ (take infinity as zero potential)
  - For a collection of charges $V = k\Sigma \frac{Q_i}{r_i}$
  - Equipotential
    - Equipotential surface is perpendicular to the electric field
    - On a equipotential surface $\Delta V = 0$
  - Potential gradient
    - $E = -\frac{\mathrm{d}V}{\mathrm{d}r}$
    - In 3-D, $\vec{E} = -\nabla \vec{V} = -\frac{\partial V}{\partial x}\vec{\imath} - \frac{\partial V}{\partial y}\vec{\jmath} - \frac{\partial V}{\partial z}\vec{k}$
  - In general, V is continuous
    - For a sphere $V_{inside} = V_{sphere}$
- Ensemble of point charges
  - The electrical potential energy of a system of fixed point charges is equal to the work that must be done by an external agent to assemble the system (bringing each charge from infinity to its final position)
  - For 3 charges: $U = k\frac{q_1 q_2}{r_{12}} + k\frac{q_1 q_3}{r_{13}} + k\frac{q_2 q_3}{r_{23}}$
  - For N charges: $U = k\sum_{\mathrm{i=1,j>i}}^{N} \frac{q_i q_j}{r_{ij}} = \frac{1}{2}k\sum_{\mathrm{i=1,j=1}}^{N} \frac{q_i q_j}{r_{ij}}$

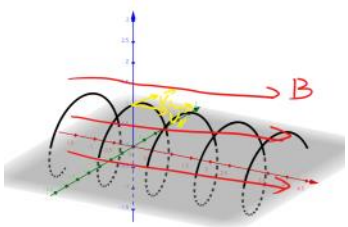
- Capacitance and capacitors
  - Calculate the capacitance
    - Calculate E due to charge using Gauss's Law
    - Calculate potential difference from E
    - Calculate $C = \frac{Q}{\Delta V}$
  - Parallel plate capacitor



    - $U = \frac{1}{2}\frac{Q^2}{C} = \frac{Q^2 x}{2A\varepsilon_0}$

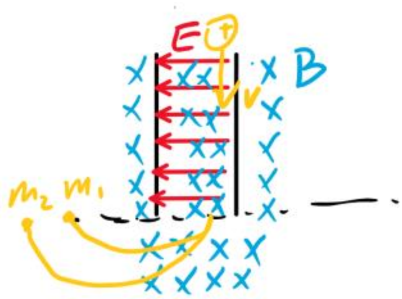
        - $F_x = -\frac{dU}{dx} = -\frac{Q^2}{2A\varepsilon_0}$
    - Dielectric
        - Dielectric reduces the potential difference across the capacitor
        - Adding an electric field make the molecule more polar and align with the field
        - Induced field is always smaller than original field $E_{induced} = \frac{E_0}{K_e}$
        - $K_e > 1$ is the dielectric constant of the dielectric material

- ## Magnetic field
    - There are no magnetic charges(monopoles)
    - The most basic magnetic field is a dipole
        - So $\phi_B = \oiint \vec{B} \cdot d\vec{A} = 0$ for closed surface
    - Magnetic field lines are continuous
    - Units for magnetic field: T(esla)
- ## Magnetic force on charged particle
    - $\vec{F} = q\vec{v} \times \vec{B}$
    - If an electric field is also present : $\vec{F} = q\vec{v} \times \vec{B} + q\vec{E}$(Lorentz Force Law)
- ## Electrons in magnetic fields
    - Helical motion



        - $\overrightarrow{v_{total}} = \overrightarrow{v_\perp} + \overrightarrow{v_\parallel}$
        - $\overrightarrow{v_\perp}$ causes the circular motion
    - Cyclotron motion

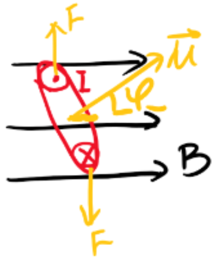
        - $R = \frac{mv}{qB}$
        - $\omega = \frac{v}{R} = \frac{qB}{m}$
    - Velocity selector



        - Particles can go through the selector if and only if $\overrightarrow{F_e} = \overrightarrow{F_B}$
        - The selected velocity $v = \frac{E}{B}$
        - Magnetic field separates particles by mass, the greater the mass, the larger the radius

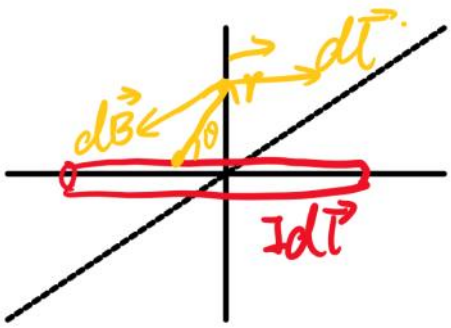
- $R = \frac{mv}{qB}$

- Magnetic moment
  - It makes the closed circuit inside a magnetic field rotate
  - $\tau = BIA \sin\varphi$
  - $\vec{\mu} = I\vec{A} = IA\vec{n}$
  - Magnetic dipole moment $\vec{\tau} = \vec{\mu} \times \vec{B}$
  - Magnetic dipole potential $U = -\vec{\mu} \cdot \vec{B}$

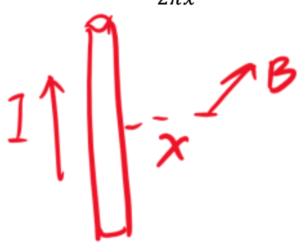


  - Application: direct current electric motor
    - Remain the force/torque direction by switching the current direction

- Source of magnetic field
  - Biot-Savart Law:
    - $d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\, d\vec{l} \times \vec{r}}{r^3} = \frac{\mu_0}{4\pi}\frac{I\, d\vec{l} \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{I\, dl\, \sin\theta}{r^2}$
      - $\mu_0 = 4\pi \times 10^{-7}\, N/A^2$
    - It determines magnetic field $d\vec{B}$ produced at a distance from a current element $I\, d\vec{l}$



  - For a long wire $B = \frac{\mu_0 I}{2\pi x}$



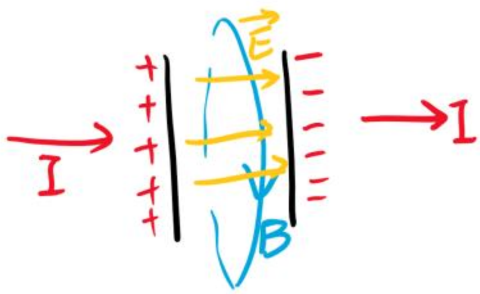
  - For a solenoid $B = \mu_0 nI$, n is the number of turns per unit length

  - Ampere's Law
    - $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$
    - Closed path with an enclosed area of current

        - Requires symmetry

- Faraday's Law
    - $\varepsilon = \oint \vec{E} \cdot \mathrm{d}\vec{l} = -\frac{\mathrm{d}\phi}{\mathrm{d}t} = -\frac{\mathrm{d}}{\mathrm{d}t} BAcos\varphi$

    - A changing magnetic flux creates an induced E-field and an induced EMF
    - Lenz's law: The changing magnetic flux generates an induced current which then creates and induces magnetic field which, in turn, tries to maintain the status quo situation
    - Induced E-field is a non-electrostatic field, unlike the field produced by charges
- Displacement current
    - $i_{\mathrm{disp}} = \varepsilon_0 \frac{\mathrm{d}\phi_E}{\mathrm{d}t}$
    - Fictitious current between two charged capacitor plates



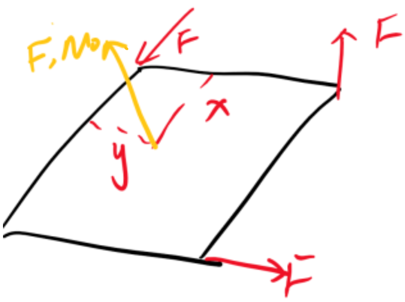
    - Apply this to Ampere's law: $\oint \vec{B} \cdot \mathrm{d}\vec{l} = \mu_0 I_{encl} + \mu_0 \varepsilon_0 \frac{\mathrm{d}\phi_E}{\mathrm{d}t}$
- Maxwell's equations
    - $\int_S \vec{E} \cdot \mathrm{d}\vec{A} = \frac{Q_{encl}}{\varepsilon_0} = 4\pi \mathrm{k} Q_{encl}$
    - $\int_S \vec{B} \cdot \mathrm{d}\vec{A} = 0$
    - $\int_C \vec{B} \cdot \mathrm{d}\vec{l} = \mu_0 I_{encl} + \mu_0 \varepsilon_0 \frac{\mathrm{d}\phi_E}{\mathrm{d}t}$
    - $\int_c \vec{E} \cdot \mathrm{d}\vec{l} = -\frac{\mathrm{d}\phi_B}{dt}$
    - $\phi_E = \int \vec{E} \cdot \mathrm{d}\vec{A}$
    - $\phi_B = \int \vec{B} \cdot \mathrm{d}\vec{A}$
    - $\mu_0 \varepsilon_0 = \frac{1}{c^2}$

- S is a closed surface, C is closed curve, c is the speed of light

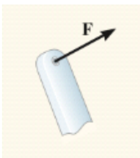
Mechanics

2019年7月4日 17:09

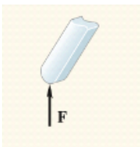
- **Statics**
- Force and Moment
  - Forces and moments are vectors
  - $\overrightarrow{M_O} = \vec{r} \times \vec{F}$
  - $|M| = |F||d|, d = r sin\theta$
  - Moment about an axis: $\overrightarrow{M_{OA}} = \overrightarrow{u_{OA}} \cdot \overrightarrow{M_O}$
  - Wrench (where $\vec{F}$ is parallel with $\overrightarrow{M_0}$)



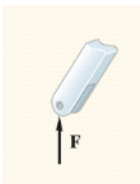
  - Joint connections
    - Cable



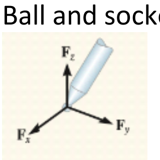
    - Smooth surface



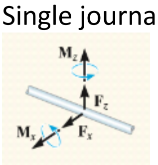
    - Roller



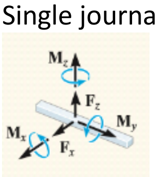
    - Ball and socket



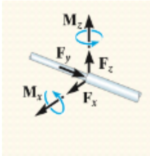
    - Single journal bearing



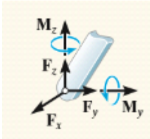
    - Single journal bearing with square shaft



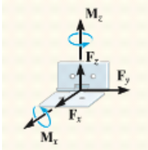
    - Single thrust bearing



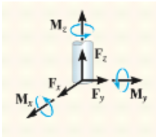
        - Single smooth pin



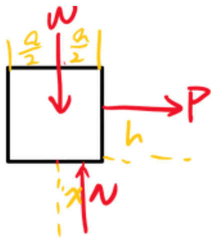
        - Single hinge



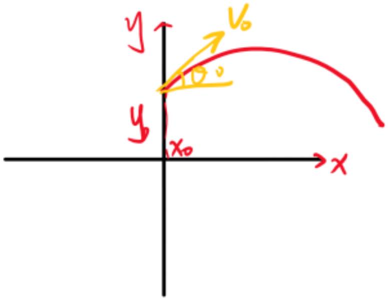
        - Fixed support



- Impending motion
    - Sliding $f = \mu_S N$
        - Angle of friction $\varphi = \arctan \mu$
    - Tipping $Wx = Ph, x = \frac{a}{2}$



- Dynamics
- Projectile motion
    - Trajectory equation: $y(x) = a(x - x_0)^2 + b(x - x_0) + y_0$
        - $a = -\dfrac{g}{2v_0^2 \cos^2 \theta_0}$
        - $b = \tan \theta_0$

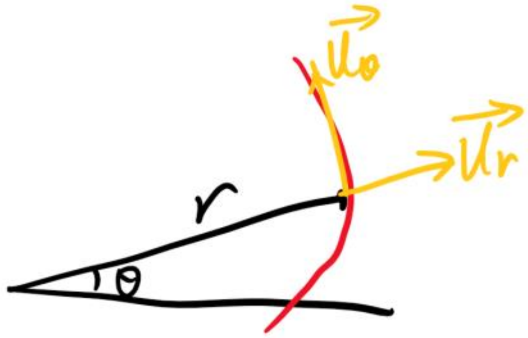


- Circular motion

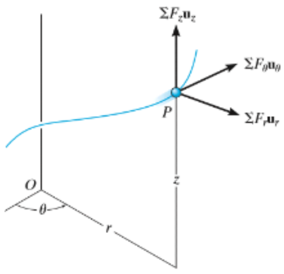
    - $\vec{v} = v\overrightarrow{u_t}, v = \dfrac{ds}{dt}$

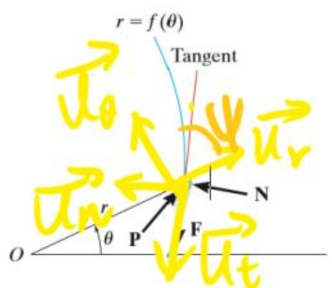
- $\vec{a} = a_t\overrightarrow{u_t} + a_n\overrightarrow{u_n}, a_t = \frac{dv}{dt} = v\frac{dv}{ds}, a_n = \frac{v^2}{r}, a = \sqrt{a_n^2 + a_t^2}$
- On a circular track
    - at distance s along the track after passing a point, given $a_t = \frac{s}{\tau^2}$
        - $v(s) = \frac{1}{\tau}\sqrt{s^2 + (v_0\tau)^2}$
        - $t = \tau \ln\left(u + \sqrt{u^2+1}\right), u = \frac{s}{v_0\tau}$
    - At time t after passing a point
        - $s(t) = v_0\tau\sinh\left(\frac{t}{\tau}\right)$
        - $v(t) = v_0\cosh\left(\frac{t}{\tau}\right)$
        - $\sinh(x) = \frac{e^x - e^{-x}}{2}, \cosh(x) = \frac{e^x + e^{-x}}{2}$

- Curvilinear motion



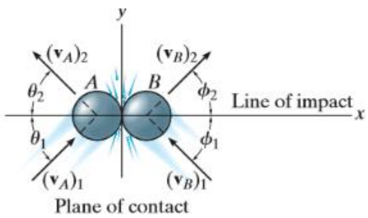
    - $v_r = \frac{dr}{dt}, v_\theta = r\frac{d\theta}{dt}, v = \sqrt{v_r^2 + v_\theta^2}$
    - $a_r = \frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2, a_\theta = r\frac{d^2\theta}{dt^2} + 2\frac{dr}{dt}\frac{d\theta}{dt}, \quad a = \sqrt{a_r^2 + a_\theta^2}$
- Dependent motion(Pulley)
    - A has n strings and B has m strings, then $nv_A = m\,v_b$
- Relative motion
    - $\overrightarrow{v_B} = \overrightarrow{v_A} + \overrightarrow{v_{B/A}}$
- Force related to curvilinear motion (cylindrical coordinates)





- $\tan \psi = \frac{r}{dr/\mathrm{d}\theta}$
- Everything in curvilinear motion applies
- To find F and N, project a_r, a_θ onto $\overrightarrow{u_t}$ and $\overrightarrow{u_n}$ direction, then use $F_{net} = ma$ on the direction

- Work and energy
  - Work: $U_{1-2} = \int_{r_1}^{r_2} \vec{F} \cdot \mathrm{d}\vec{r}$
  - $T = \frac{1}{2}mv^2$
  - $V = mgh + \frac{1}{2}k\Delta x^2$
  - Conservation of energy: $\Sigma T_1 + \Sigma V_1 + U_{1-2} = \Sigma T_2 + \Sigma V_2$
- Linear impulse and momentum
  - Conservation of momentum of an object: $mv_1 + \int_{t_1}^{t_2} F\,\mathrm{d}t = mv_2$
  - Impact



    - Coefficient of restitution: $\mathrm{e} = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$
    - $m_A(v_A)_1 + m_B(v_B)_1 = m_A(v_A)_2 + m_B(v_B)_2$
      - Only the normal velocity changes, the tangent velocity do not change
      - Velocity has directions, "-"sign means opposite direction
- Angular momentum
  - $\overrightarrow{H_O} = \vec{r} \times m\vec{v}$
  - $\Sigma\left(\overrightarrow{H_0}\right)_1 + \int_{t_1}^{t_2} \overrightarrow{M_0} dt = \Sigma\left(\overrightarrow{H_0}\right)_2$